Multiply to make equivalent fractions

You cut every piece of a cake into smaller pieces. The cake did not change, so what happens to the fraction?

WORDS TO KNOW

Numerator: the top number of a fraction. It counts the pieces you have.

Denominator: the bottom number. It counts how many equal pieces make the whole.

Equivalent fractions: two fractions that name the same amount of the same whole.

Counting number: one of 1, 2, 3, 4 and so on.

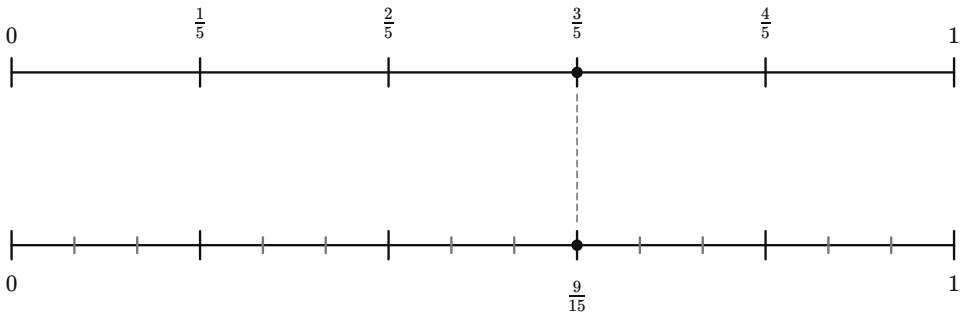
Family: all the fractions you get from one fraction by cutting its pieces into 2, 3, 4, and so on.

KEY IDEAS

1. Cut every piece the same way

Mark $\frac{3}{5}$ on a number line: 3 jumps of one fifth. Now cut every fifth into 3 smaller jumps. The space from 0 to 1 now holds 5 × 3 = 15 small jumps, and the dot is 3 × 3 = 9 small jumps from 0. The dot did not move. Only the ruler got finer, so $\frac{3}{5} = \frac{9}{15}$.

The cut goes through **every** piece: the pieces you count and the pieces that make the whole. So both numbers change together. The new pieces are smaller, but there are more of them.



2. The rule: multiply the top and the bottom by the same number

Multiply the top and the bottom by the same counting number. You get an equivalent fraction.

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

Multiplying both by 3 is the same as multiplying by $\frac{3}{3}$, which is 1. Times 1 never changes a number.

The rule works for fractions bigger than one too.

3. Missing numbers: find the cut first

Use the pair of numbers you know to find the cut. Then do the same cut to the other number.

Each fraction has an endless family. The tops count by the old top and the bottoms count by the old bottom.

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WATCH OUT

- **Do not add.** Adding 2 to the top and the bottom of $\frac{3}{4}$ gives $\frac{5}{6}$, a different amount. Only multiplying by the same number keeps the amount.
- **Use the same number on top and bottom.** One cut goes through every piece. $\frac{1}{4}$ is not $\frac{2}{12}$: the bottom was multiplied by 3 but the top by 2.
- **Only multiples of the bottom.** $\frac{5}{6}$ has no name in eighths, because no counting number times 6 makes 8.
- **The same whole.** Equivalent fractions name the same part of the same whole. Half of a big pizza is more pizza than half of a small one.

WORKED EXAMPLES

Example 1. A tray of cornbread has 5 equal strips, and 4 strips have cheese. Every strip is cut into 3 squares. Write the cheese part as two equivalent fractions.

1. Before the cut, 4 of 5 strips have cheese: $\frac{4}{5}$.
2. The whole: 5 strips, each now 3 squares, so 5 × 3 = 15 squares.
3. The cheese: 4 strips, each now 3 squares, so 4 × 3 = 12 squares.

Answer: $\frac{4}{5} = \frac{12}{15}$. **Check:** $\frac{4\times3}{5\times3} = \frac{12}{15}$.

Example 2. Find each missing number: $\frac{5}{4} = \frac{?}{12}$ and $\frac{6}{5} = \frac{12}{?}$.

1. First one: the bottom tells the cut. 4 pieces became 12, and 4 × 3 = 12, so every quarter was cut into 3.
2. Do the same to the top: 5 × 3 = 15.
3. Second one: the top tells the cut. 6 pieces became 12, and 6 × 2 = 12, so every fifth was cut into 2.
4. Do the same to the bottom: 5 × 2 = 10.

Answer: $\frac{5}{4} = \frac{15}{12}$ and $\frac{6}{5} = \frac{12}{10}$.

Example 3. A trail is $\frac{9}{5}$ miles long. Write the length in tenths and in hundredths.

1. To turn fifths into tenths, cut every fifth into 2: 5 × 2 = 10. So the top is 9 × 2 = 18.
2. To turn tenths into hundredths, cut every tenth into 10: 10 × 10 = 100. So the top is 18 × 10 = 180.

Answer: $\frac{9}{5} = \frac{18}{10} = \frac{180}{100}$ miles. **Check:** in one step, 5 × 20 = 100 and 9 × 20 = 180.

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