Other number bases

Our numbers bundle in tens. What if you bundled straws in fives instead, or in twos like a computer does? Place value works the same way with any bundle size.

WORDS TO KNOW

Base: the bundle size. It can be any whole number 2 or more. Our usual numbers are base ten.

Base two (binary): places worth 1, 2, 4, 8, 16 and so on. The digits are only 0 and 1.

Expanded form: each digit times the value of its place, all added up.

Base five: places worth 1, 5, 25, 125 and so on. The digits are 0, 1, 2, 3 and 4.

Base name, as in 23_{five}: the small word after the digits names the base. A number with no small word is in base ten.

KEY IDEAS

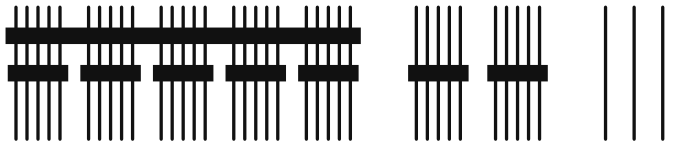
1. Bundle in fives

Chayton puts a band around every five straws. Five bundles get a bigger band: one big bundle is 5 × 5 = 25 straws. Then he writes how many big bundles, bundles and loose straws he has.

In base five each place is worth five of the place to its right:

$$\ldots \quad 125 \quad 25 \quad 5 \quad 1$$

To find the value, use expanded form: $123_{\text{five}} = 1 \times 25 + 2 \times 5 + 3 = 38$.



1 twenty-five 2 fives 3 ones

1 2 3

2. A digit is always less than the base

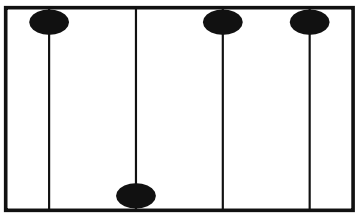
If a place held five or more in base five, five of them would be tied into one of the next place. So every digit is 0, 1, 2, 3 or 4. In base b the digits go from 0 up to one less than b.

In every base, 10 means one bundle and no ones, so it is the base itself: $10_{\text{five}} = 5$ and $10_{\text{two}} = 2$.

3. Base two uses only 0 and 1

In base two each place is worth twice the place to its right. A computer stores numbers this way, with on and off. On a base two abacus a bead pushed up counts its place, and a bead down counts nothing.

To read a binary number, add up the places that hold a 1. The abacus shows $1011_{\text{two}} = 8 + 2 + 1 = 11$.



8 4 2 1

1 0 1 1

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WATCH OUT

- **No digit can be as big as the base.** 52_{five} is not a base five number, because a place cannot hold 5.
- **Zeros hold places, in any base.** 102_{five} and 12_{five} are different numbers.
- **The same digits can name different numbers.** $11_{\text{two}} = 3$, but $11_{\text{five}} = 6$ and 11 in base ten is eleven. Always check the base.

WORKED EXAMPLES

Example 1. Write 240_{five} in base ten.

1. Read the places from the right: ones, fives, twenty-fives.
2. There are 2 twenty-fives, 4 fives and 0 ones.
3. Expanded form: $2 \times 25 + 4 \times 5 + 0 = 50 + 20 + 0$.

Answer: $240_{\text{five}} = 70$.

Example 2. Write 105 in base five.

1. The places are 1, 5, 25, 125. Since 125 is more than 105, start at 25.
2. Four twenty-fives make 100, and five would be too many. That leaves $105 - 100 = 5$.
3. One five fits in 5, leaving 0 loose ones. The ones place gets a 0.

Answer: $105 = 410_{\text{five}}$. **Check:** $4 \times 25 + 1 \times 5 + 0 = 100 + 5 = 105$.

Example 3. Write 26 in base two. Then write 1110_{two} in base ten.

1. The places are 1, 2, 4, 8, 16, 32. Since 32 is too big, start at 16.
2. Take 16, leaving 10. Take 8, leaving 2. Skip 4. Take 2, leaving 0. Skip 1.
3. Write 1 for each place taken and 0 for each place skipped, biggest place first: 11010_{two}.
4. For 1110_{two}, the places with a 1 are 8, 4 and 2: $8 + 4 + 2 = 14$.

Answer: $26 = 11010_{\text{two}}$ and $1110_{\text{two}} = 14$. **Check:** $16 + 8 + 2 = 26$.

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