Powers of ten

A billion is written 1,000,000,000, and one missed zero makes it ten times too small. A power of ten gives numbers like this a short name that tells you how many times ten was multiplied.

WORDS TO KNOW

Power of ten: the number you get by starting at 1 and multiplying by 10 some whole number of times.

Ten squared, ten cubed: 10^2 is called ten squared, and 10^3 is called ten cubed.

10^n**:** read "ten to the n-th power". The 10 is the **base**. The small raised number n is the **exponent**: it counts how many times you multiply by 10.

Expanded form: a number written as each digit times the value of its place, all added up.

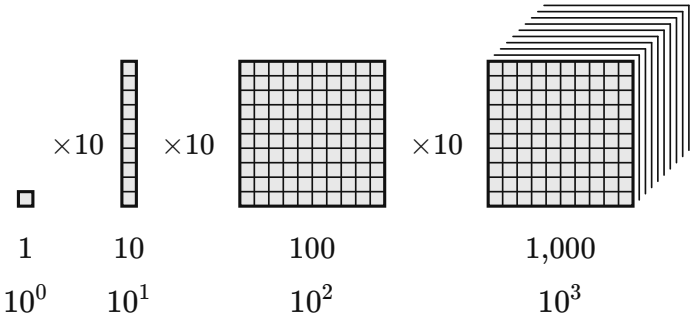
KEY IDEAS

1. The exponent counts the tens

Each base-ten block is ten of the block before it. The exponent is how many "times 10" steps you take from the single square.

$$10^n = 1 \times \underbrace{10 \times 10 \times \cdots \times 10}_{n \text{ tens}}$$

Written out, 10^n is a 1 followed by n zeros. For example, 10^{12} is a 1 followed by twelve zeros.



2. Ten to the zero is one

Start at 1 and multiply by 10 zero times. Nothing changes, so you still have 1. The pattern agrees: each step down in the exponent divides by 10.

One step down from $10^1 = 10$ gives

$$10^0 = 10 \div 10 = 1$$

3. Every place is a power of ten

The ones place is 10^0. Each step to the left is one more factor of 10: tens are 10^1, hundreds 10^2, thousands 10^3. A digit tells how many of that power the number has.

A whole number is the sum of each digit times the power of ten for its place. A term with the digit 0 is worth nothing, so it may be left out of the sum, but the 0 must stay in the written number.

$$6{,}203 = 6 \times 10^3 + 2 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$$

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4. Multiplying a whole number by a power of ten

Each factor of 10 moves every digit one place left and puts a 0 in the ones place.

> To multiply a whole number by 10^n, write n zeros after it.

WATCH OUT

- **The exponent does not mean "times".** 10^{15} is not 10 × 15 = 150. It is fifteen tens multiplied together, a 1 followed by fifteen zeros.
- 10^0 **is 1, not 0.** Doing no multiplying leaves the 1 you started with.
- **Zeros already in the number stay.** 60×10^3 is 60,000, with four zeros: the 0 in 60 plus three new ones.
- **The zeros rule is for whole numbers.** Putting a 0 at the end of a decimal does not change it: 3.6 and 3.60 are equal.

WORKED EXAMPLES

Example 1. A number is written as a 1 followed by eleven zeros. Write it with commas, and as a power of ten.

1. Write the 1, then the zeros in groups of three from the right: 100,000,000,000.
2. Each zero after the 1 comes from one factor of 10. Eleven zeros means eleven factors of 10.
3. The exponent counts the factors of 10, so it is 11.

Answer: $100{,}000{,}000{,}000 = 10^{11}$. **Check:** three groups of three zeros make nine, and the two zeros in front of them make eleven.

Example 2. Write 40,382 in expanded form using powers of ten.

1. Count places from the ones place, which is 10^0. The 4 is four places left, so it is 4×10^4.
2. The 0 is in the thousands place, 0×10^3. The 3, 8 and 2 give 3×10^2, 8×10^1 and 2×10^0.
3. Add the terms, and leave out the zero term because it is worth nothing.

Answer: $40{,}382 = 4 \times 10^4 + 3 \times 10^2 + 8 \times 10^1 + 2 \times 10^0$. **Check:** 40,000 + 300 + 80 + 2 = 40,382.

Example 3. Find (a) 306×10^3 **and (b)** $7{,}020 \times 10^2$**.**

1. (a) The exponent is 3, so write three zeros after 306. The 0 inside 306 stays where it is.
2. (b) The exponent is 2, so write two zeros after 7,020. Its own zeros stay too.

Answer: (a) 306,000 (b) 702,000. **Check:** in (b), 7,020 × 10 = 70,200, then 70,200 × 10 = 702,000.

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