Grade 5 Unit 1 worksheet: Numbers in other bases (answer key)

ANSWERS AT A GLANCE

1. (a) 21 (b) 53 (c) 152 (d) 141_{five} (e) 401_{five} (f) 1003_{five}

2. (a) 9 (b) 19 (c) 56 (d) 10101_{two} (e) 100101_{two} (f) 110010_{two}

3. (a) 4 cases, 2 boxes and 2 loose crayons. (b) 422_{five}

4. 43_{five}, 24, 11001_{two} (they are 23, 24 and 25).

5. 40_{five}, 42_{five}, 43_{five}, 100_{five}

6. (a) base five (b) base two (c) base ten

WORKED SOLUTIONS

1. Write (a) to (c) in base ten. Write (d) to (f) in base five.

(a) 41_{five} (b) 203_{five} (c) 1102_{five} (d) 46 (e) 101 (f) 128

ANSWER

(a) 21 (b) 53 (c) 152 (d) 141_{five} (e) 401_{five} (f) 1003_{five}

SOLUTION

In base five the places from the right are worth 1, 5, 25 and 125.

(a) $41_{\text{five}} = 4 \times 5 + 1 = 21$.

(b) $203_{\text{five}} = 2 \times 25 + 0 \times 5 + 3 = 50 + 3 = 53$.

(c) $1102_{\text{five}} = 1 \times 125 + 1 \times 25 + 0 \times 5 + 2 = 125 + 25 + 2 = 152$.

For (d) to (f), start with the biggest place that fits and take as many of it as you can.

(d) One twenty-five fits in 46, leaving 21. Four fives fit in 21, leaving 1. So $46 = 1 \times 25 + 4 \times 5 + 1 = 141_{\text{five}}$.

(e) Four twenty-fives make 100, leaving 1. No five fits in 1, so the fives place holds 0. So $101 = 4 \times 25 + 0 \times 5 + 1 = 401_{\text{five}}$.

(f) One hundred twenty-five fits in 128, leaving 3. No twenty-five and no five fit in 3. So $128 = 1 \times 125 + 0 \times 25 + 0 \times 5 + 3 = 1003_{\text{five}}$. The two zeros hold their places open.

2. Write (a) to (c) in base ten. Write (d) to (f) in base two.

(a) 1001_{two} (b) 10011_{two} (c) 111000_{two} (d) 21 (e) 37 (f) 50

ANSWER

(a) 9 (b) 19 (c) 56 (d) 10101_{two} (e) 100101_{two} (f) 110010_{two}

SOLUTION

In base two the places from the right are worth 1, 2, 4, 8, 16 and 32.

(a) $1001_{\text{two}} = 8 + 1 = 9$.

(b) $10011_{\text{two}} = 16 + 2 + 1 = 19$.

(c) $111000_{\text{two}} = 32 + 16 + 8 = 56$.

For (d) to (f), take the biggest place that fits, then keep going. Write 1 for each place taken and 0 for each place skipped.

Free sample from math.wiki for classroom use. Do not post. math.wiki

(d) 21 = 16 + 4 + 1, so $21 = 10101_{\text{two}}$.

(e) 37 = 32 + 4 + 1, so $37 = 100101_{\text{two}}$.

(f) 50 = 32 + 16 + 2, so $50 = 110010_{\text{two}}$.

3. Mateo's class packs crayons for a craft fair. They put 5 crayons in a box and 5 boxes in a case. They have 112 crayons.

(a) How many full cases, extra boxes and loose crayons do they get?

(b) Write 112 in base five.

ANSWER

(a) 4 cases, 2 boxes and 2 loose crayons. (b) 422_{five}

SOLUTION

(a) A case holds 5 × 5 = 25 crayons. Four cases hold 4 × 25 = 100 crayons, and five would need 125, too many. That leaves 112 − 100 = 12 crayons. Two boxes hold 10, leaving 2 loose crayons.

(b) Cases are the twenty-fives place, boxes the fives place and loose crayons the ones place:

$$112 = 4 \times 25 + 2 \times 5 + 2 = 422_{\text{five}}.$$

4. Put these numbers in order from least to greatest.

$$43_{\text{five}} \qquad 11001_{\text{two}} \qquad 24$$

ANSWER

43_{five}, 24, 11001_{two} (they are 23, 24 and 25).

SOLUTION

Write each one in base ten first.

$43_{\text{five}} = 4 \times 5 + 3 = 23$.

$11001_{\text{two}} = 16 + 8 + 1 = 25$.

24 has no small base word, so it is already in base ten.

From least to greatest: 43_{five}, 24, 11001_{two}. Looking at the digits alone would give the wrong order.

5. These numbers count up by one, all written in base five. Fill in the blanks.

$$33_{\text{five}},\ 34_{\text{five}},\ _____,\ 41_{\text{five}},\ _____,\ _____,\ 44_{\text{five}},\ _____$$

ANSWER

40_{five}, 42_{five}, 43_{five}, 100_{five}

SOLUTION

In base five the digits only go up to 4. After 34_{five} the ones place is full, so it rolls over to 0 and the fives place goes up by one: 40_{five}. Then 41_{five}, 42_{five}, 43_{five}, 44_{five}.

Free sample from math.wiki for classroom use. Do not post. math.wiki

After 44_{five} both places are full. The ones roll over, then the fives roll over, and the twenty-fives place gets a 1: 100_{five}.

Check in base ten: the count is 18, 19, 20, 21, 22, 23, 24, 25, and $100_{\text{five}} = 25$.

6. Each numeral below names the number thirty. One is in base two, one is in base five and one is in base ten. Which is which? Show how you know.

(a) 110 (b) 11110 (c) 30

ANSWER

(a) base five (b) base two (c) base ten

SOLUTION

Try each base and see which one gives 30.

(a) In base five, $110_{\text{five}} = 1 \times 25 + 1 \times 5 + 0 = 30$. (In base two it would be only 4 + 2 = 6.)

(b) In base two the places are 16, 8, 4, 2, 1, so $11110_{\text{two}} = 16 + 8 + 4 + 2 = 30$.

(c) 30 has a digit 3, so it cannot be base two. In base five it would be 3 × 5 = 15. So it is base ten: three tens and no ones.

Free sample from math.wiki for classroom use. Do not post. math.wiki