Adding and subtracting fractions with unlike denominators

Half an hour at the barre and three tenths of an hour on jumps. How do you add two fractions whose pieces are different sizes?

WORDS TO KNOW

Denominator: the bottom number. It names the size of the pieces, not how many you have.

Common denominator: a number both denominators divide into evenly, so both fractions can be counted in one piece size.

Simplest form: no whole number bigger than 1 divides both the top and the bottom.

Unlike denominators: bottom numbers that are different, such as 3 and 4.

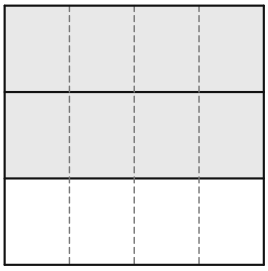
Least common denominator: the smallest common denominator.

Mixed number: a whole number and a fraction together, such as $1\frac{3}{10}$.

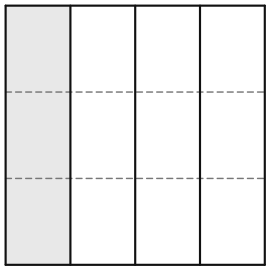
KEY IDEAS

1. The pieces must match before you count

You cannot add 2 thirds and 1 quarter as "3" of anything. Cut a square into 3 rows and 4 columns. Every cell is one twelfth, and both fractions can be counted in cells: $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$. So the sum is $\frac{11}{12}$.



$\frac{2}{3} = \frac{8}{12}$



$\frac{1}{4} = \frac{3}{12}$

2. Rename, then add or subtract the tops

Step 1. Find a common denominator. If the larger denominator is a multiple of the smaller, use it, and only one fraction changes. If not, list multiples of the larger one until the smaller one divides it. The product of the denominators always works too.

Step 2. Rename: multiply the top and the bottom of each fraction by the same number.

Step 3. Add or subtract the tops. Keep the denominator.

$$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$$

Step 4. Simplify. Write a sum greater than 1 as a mixed number.

The denominator stays the same because the pieces are still the same size. You have only counted how many of them there are.

3. Sums greater than one

Two fractions less than 1 can add up to more than 1. When the top is bigger than the bottom, divide the top by the bottom. The quotient is the whole number and the remainder is the new top.

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WATCH OUT

- **Never add the bottoms.** $\frac{1}{2} + \frac{3}{10}$ is not $\frac{4}{12}$. That is less than $\frac{1}{2}$ alone, so it cannot be the sum.
- **Change the top too.** When you rename, multiply the top by the same number as the bottom. Changing only the bottom changes the amount.
- **Larger minus smaller.** In this lesson you always take the smaller fraction from the larger one.
- **Check the size.** Round each fraction to 0, $\frac{1}{2}$ or 1. The answer should be close to the rounded sum.

WORKED EXAMPLES

Example 1. Find $\frac{1}{4} + \frac{7}{20}$ in simplest form.

1. $20 \div 4 = 5$, a whole number, so 20 is a multiple of 4. Use twentieths, and change only $\frac{1}{4}$: multiply its top and bottom by 5.
$$\frac{1}{4} + \frac{7}{20} = \frac{5}{20} + \frac{7}{20} = \frac{12}{20}$$
2. Both 12 and 20 can be divided by 4, so $\frac{12}{20} = \frac{3}{5}$.

Answer: $\frac{3}{5}$. **Check:** $\frac{7}{20}$ is a bit less than $\frac{10}{20}$, which is $\frac{1}{2}$. So the sum is a bit less than $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$, and $\frac{3}{5}$ is.

Example 2. Find $\frac{8}{15} - \frac{3}{10}$ in simplest form.

1. Neither 15 nor 10 divides the other. The product 150 would work, but look for a smaller one. Multiples of 15: 15, 30. 10 divides 30, so use 30.
2. Multiply $\frac{8}{15}$ by $\frac{2}{2}$ and $\frac{3}{10}$ by $\frac{3}{3}$.
$$\frac{8}{15} - \frac{3}{10} = \frac{16}{30} - \frac{9}{30} = \frac{7}{30}$$
3. 7 and 30 share no factor except 1, so $\frac{7}{30}$ is in simplest form.

Answer: $\frac{7}{30}$. **Check:** add back. $\frac{7}{30} + \frac{9}{30} = \frac{16}{30}$, which is $\frac{8}{15}$.

Example 3. Find $\frac{5}{7} + \frac{1}{2}$. Write the answer as a mixed number.

1. 7 and 2 share no factor except 1, so use $7 \times 2 = 14$.
$$\frac{5}{7} + \frac{1}{2} = \frac{10}{14} + \frac{7}{14} = \frac{17}{14}$$
2. The top is bigger than the bottom. $17 \div 14 = 1$ remainder 3, so $\frac{17}{14} = 1\frac{3}{14}$.

Answer: $1\frac{3}{14}$. **Check:** $\frac{5}{7}$ is between $\frac{1}{2}$ and 1, so adding $\frac{1}{2}$ must give between 1 and $1\frac{1}{2}$. It does.

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