Volume of a box

You can measure the three edges of a box with a ruler. Is that enough to know how much it holds, without filling it?

WORDS TO KNOW

Rectangular prism (box): a solid with six flat rectangle faces that meet at square corners, like a shoebox or a brick.

Base: the face the box rests on. Its area is written B.

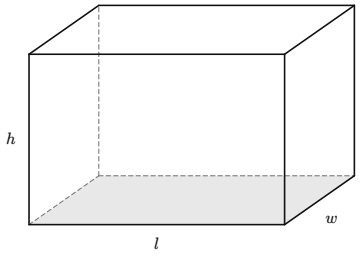
Length, width, height (l, w, h): the three kinds of edge of a box.

Volume (V): the number of unit cubes that fill the box. It is in cubic units, such as cm^3 or ft^3.

KEY IDEAS

1. Length times width times height

Fill a box with unit cubes. The floor is l by w, so one layer holds $l \times w$ cubes. The box is h units tall, so h layers fit. The rule is a fast way to count all the cubes.



$$V = l \times w \times h$$

All three edges must be in the same unit. The answer is in cubic units.

2. Base area times height, and any face can be the base

The first two factors, $l \times w$, are the area of the base. So the same rule can be written with the base area B. This helps when you know the floor area but not its two sides.

$$V = B \times h$$

You may multiply the three edges in any order and group them any way. The answer is the same. So pick the pair that is easiest, such as one that makes 10 or 100.

Turning a box onto another face does not change the space inside. Whichever face it rests on is the base, and the edge standing up is the height.

3. Finding a missing edge

Division undoes multiplication. Each unit of height is one layer of B cubes, so the height is how many layers of B make V.

Height $= V \div B$. Base area $= V \div h$.

Know the volume and two edges? Multiply the two edges, then divide the volume by that product.

WATCH OUT

- **Put all three edges in one unit first.** Feet times inches counts no real cube. Change one unit before you multiply.
- **What a box holds comes from its inside edges.** Thick walls make the outside bigger than the inside.
- **The same volume can come in very different shapes.** A long low box and a tall narrow box can hold the same amount, but a long object may fit in only one of them.
- **A leftover still needs a box.** If you need boxes for a load and some is left over, you need one more box, so round up.

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WORKED EXAMPLES

Example 1. A box is 13 cm long, 7 cm wide and 3 cm tall. What is its volume?

1. All edges are in centimeters, so the volume is in cubic centimeters.
2. The floor: 13 × 7 = 91. That is 91 centimeter cubes in one layer.
3. The box is 3 cm tall, so 3 layers fit: 91 × 3 = 273.

Answer: 273 cm^3. **Check:** another order, 7 × 3 = 21 and 21 × 13 = 273.

Example 2. A crate is 25 in long, 19 in wide and 4 in tall. Find its volume without a long multiplication.

1. You may multiply the edges in any order. Pick the pair that makes a round number: 25 × 4 = 100.
2. That is the crate standing on its 25 by 4 face. The edge left over is 19: 100 × 19 = 1,900.

Answer: 1,900 in^3. **Check:** in the plain order, 25 × 19 = 475, then 475 × 4 = 1,900. The same, with harder steps.

Example 3. A box holds 980 cubic inches. Its floor is 10 in by 7 in. How tall is it?

1. The base area: $B = 10 \times 7 = 70$ square inches.
2. Each inch of height is a layer of 70 inch cubes. How many layers of 70 make 980? $h = 980 \div 70 = 14$.

Answer: 14 inches. **Check:** 10 × 7 × 14 = 70 × 14 = 980.

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