Linear Algebra Unit 1 quiz 2: Span, Linear independence

Name _____ Date _____ Period _____

7 questions, 20 points

1. Let $\mathbf{v}_1 = (1, 0)$ and $\mathbf{v}_2 = (1, 1)$, and let W be the set of all $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$ with $c_1 \geq 0$ and $c_2 \geq 0$. (2 points)

(a) Give a vector in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ that is not in W, with a reason.

(b) Explain why W is not the span of any list of vectors.

2. Let $\mathbf{u}_1 = (1, 2, -1)$, $\mathbf{u}_2 = (2, 3, 1)$, $\mathbf{u}_3 = (0, 1, -3)$ and $\mathbf{u}_4 = (3, 5, 0)$. (3 points)

(a) Do these four vectors span $\mathbb{R}^3$? Use an echelon form to decide.

(b) Keep $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$, and replace $\mathbf{u}_4$ by $(3, 5, c)$. For which numbers c do the four vectors span $\mathbb{R}^3$?

3. Let $\mathbf{v}_1 = (1, 0, -1, 2)$, $\mathbf{v}_2 = (0, 1, 2, -1)$ and $\mathbf{v}_3 = (1, 1, 1, 0)$ in $\mathbb{R}^4$, and let $\mathbf{d} = (2, 1, 1, 0)$. (3 points)

(a) Find one equation in x_1, x_2, x_3, x_4 satisfied by exactly the vectors of $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and use it to show that $\mathbf{d}$ is not in the span.

(b) Changing exactly one entry of $\mathbf{d}$ can put it in the span. For each of the four entries, say what it must be changed to, or that no change of that entry works.

4. For which real numbers a and b do $(a, 2)$ and $(3, b)$ span $\mathbb{R}^2$? For every other pair a, b, describe their span by an equation. Use row reduction, not a formula. (3 points)

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5. Let $\mathbf{v}_1 = (1, -2, 0, 3)$, $\mathbf{v}_2 = (2, -3, 1, 4)$, $\mathbf{v}_3 = (0, 1, 2, -1)$ and $\mathbf{v}_4 = (1, -3, -1, 5)$ in $\mathbb{R}^4$. (3 points)

(a) Find every dependence relation among $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$.

(b) For each of the four vectors, decide whether the list of the OTHER three is linearly independent.

6. A chemist records a reaction as a vector: for the substances in the order $(\mathrm{C}, \mathrm{O_2}, \mathrm{CO}, \mathrm{CO_2})$, each entry is the number of moles produced (positive) or used up (negative). Four reactions of carbon and oxygen give - $\mathbf{r}_1 = (-1, -1, 0, 1)$ for $\mathrm{C} + \mathrm{O_2} \to \mathrm{CO_2}$, - $\mathbf{r}_2 = (-2, -1, 2, 0)$ for $2\mathrm{C} + \mathrm{O_2} \to 2\mathrm{CO}$, - $\mathbf{r}_3 = (0, -1, -2, 2)$ for $2\mathrm{CO} + \mathrm{O_2} \to 2\mathrm{CO_2}$, - $\mathbf{r}_4 = (-1, 0, 2, -1)$ for $\mathrm{C} + \mathrm{CO_2} \to 2\mathrm{CO}$. (3 points)

(a) Show that $\mathbf{r}_1, \mathbf{r}_2$ is linearly independent.

(b) Write $\mathbf{r}_3$ and $\mathbf{r}_4$ each as a linear combination of $\mathbf{r}_1$ and $\mathbf{r}_2$.

(c) Find a dependence relation among $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4$ in which all four weights are nonzero.

7. Let $\mathbf{v}_1 = (1, 2, -1, 0)$, $\mathbf{v}_2 = (-1, -1, 3, 2)$ and $\mathbf{v}_3 = (1, h, k, 4)$ in $\mathbb{R}^4$. Find all pairs of numbers h and k for which the list $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is linearly dependent, and for those values give a dependence relation. (3 points)

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