Linear Algebra Unit 1 quiz 3: Subspaces, Basis and dimension

Name _____ Date _____ Period _____

7 questions, 20 points

1. Decide whether each set is a subspace of $\mathbb{R}^3$. Give a reason for a yes and specific failing vectors for a no. (2 points)

(a) S, the set of all vectors $(t, 2t, t^3)$ for real t.

(b) T, the set of all vectors $(t^3, 2t^3, t^3)$ for real t.

2. Let

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & 5 & 0 \\ 1 & 0 & 3 & -1 \end{bmatrix}.$$

(a) Write the solution set N of $A\mathbf{x} = \mathbf{0}$ as a span.

(b) Find the numbers h and k for which $(h, k, 2, -1)$ is in N. (3 points)

3. Let $\mathbf{a} = (1, -2, 2)$, and let W be the set of vectors $\mathbf{x}$ in $\mathbb{R}^3$ with $\|\mathbf{x} + \mathbf{a}\| = \|\mathbf{x} - \mathbf{a}\|$. (3 points)

(a) Show that W is the solution set of one homogeneous linear equation in x, y, z, and so is a subspace of $\mathbb{R}^3$.

(b) Write W as a span.

4. Let P be the plane $3x + y - 2z = 0$ in $\mathbb{R}^3$, and let $\mathbf{b}_1 = (1, -1, 1)$ and $\mathbf{b}_2 = (2, 0, 3)$. (3 points)

(a) Show that $\mathbf{b}_1, \mathbf{b}_2$ is a basis of P. Check that they lie in P and are independent, and find dim P; do not check spanning directly.

(b) Find the coordinates of (−1, −3, −3) relative to $\mathbf{b}_1, \mathbf{b}_2$.

(c) Explain why (1, 0, 0) has no coordinates relative to $\mathbf{b}_1, \mathbf{b}_2$.

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5. Let V be the set of vectors $\mathbf{x}$ in $\mathbb{R}^4$ with $x_1 + x_2 - x_4 = 0$ and $x_2 - x_3 + 2x_4 = 0$. (3 points)

(a) Find a basis of V and give $\dim V$.

(b) Extend your basis to a basis of $\mathbb{R}^4$, and explain why the longer list is a basis.

6. For each real number k, let N_k be the set of solutions in $\mathbb{R}^4$ of

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 0 \\ 2x_1 + kx_2 - 2x_3 + 2x_4 &= 0. \end{aligned}$$

Find $\dim N_k$ for every value of k, and give a basis of N_k for the value of k where the dimension is largest. (3 points)

7. Let W be a subspace of $\mathbb{R}^n$ and let $\mathbf{v}$ be a vector of $\mathbb{R}^n$ that is not in W. Let $\mathbf{v} + W$ be the set of all vectors $\mathbf{v} + \mathbf{w}$ with $\mathbf{w} \in W$. Prove that $\mathbf{v} + W$ is not a subspace. (3 points)

Statements	Reasons

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