Linear Algebra Unit 1 worksheet: Basis and dimension

Name _____ Date _____ Period _____

1. Find the dimension of each subspace and give one basis of it.

(a) $\text{span}\{(2,-4,6),(-1,2,-3)\}$ in $\mathbb{R}^3$

(b) $\{(x_1,x_2,x_3,x_4,x_5) : x_1 = x_2 = x_3\}$ in $\mathbb{R}^5$

(c) $\text{span}\{(1,1,0),(0,1,1),(1,0,-1)\}$ in $\mathbb{R}^3$

(d) all vectors $(a, b, a-b, 2b)$ for real a and b, in $\mathbb{R}^4$

(e) the vectors $\mathbf{x}$ in $\mathbb{R}^3$ with $\mathbf{x}\cdot(1,2,-1) = 0$ and $\mathbf{x}\cdot(-2,-4,2) = 0$

(f) $\{\mathbf{0}\}$ in $\mathbb{R}^6$

2. Let $V = \text{span}\{\mathbf{b}_1, \mathbf{b}_2\}$ in $\mathbb{R}^4$, where $\mathbf{b}_1 = (1,-3,2,-3)$ and $\mathbf{b}_2 = (1,-2,-1,-3)$.

(a) Explain why $\mathbf{b}_1, \mathbf{b}_2$ is a basis of V, and give $\dim V$.

(b) Show that $\mathbf{x} = (5,-13,4,-15)$ is in V, and find its coordinates relative to $\mathbf{b}_1, \mathbf{b}_2$.

(c) Decide whether $\mathbf{y} = (5,-13,4,-14)$ is in V.

3. Let $\mathbf{v}_1 = (1,-1,2,0)$, $\mathbf{v}_2 = (2,1,1,3)$, $\mathbf{v}_3 = (3,0,3,3)$, $\mathbf{v}_4 = (0,3,-3,3)$, $\mathbf{v}_5 = (0,1,0,2)$, and let $V = \text{span}\{\mathbf{v}_1,\dots,\mathbf{v}_5\}$ in $\mathbb{R}^4$.

(a) Choose from the list a basis of V, and give $\dim V$.

(b) Write each vector you left out as a linear combination of your basis.

(c) For each of $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$, decide whether adding it to your basis of V gives a basis of $\mathbb{R}^4$.

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4. Let N be the set of solutions in $\mathbb{R}^5$ of the system

$$\begin{aligned} x_1 + x_2 - 2x_3 + 3x_5 &= 0 \\ x_1 + 2x_2 - 3x_3 + 2x_4 + x_5 &= 0. \end{aligned}$$

(a) Find a basis of N with one vector for each free variable, and give $\dim N$.

(b) Show that $\mathbf{x} = (3, 2, 4, 2, 1)$ is in N, and find its coordinates relative to your basis.

(c) Explain why, relative to this basis, the coordinates of every vector of N can be read off from three of its entries, and say which three.

5. Decide whether each statement is true or false. For a true statement, name the result that gives it; for a false one, give a counterexample with actual vectors.

(a) Any three linearly independent vectors in $\mathbb{R}^4$ span $\mathbb{R}^4$.

(b) If V is a subspace of $\mathbb{R}^5$ with $\dim V = 3$, then any four vectors in V are linearly dependent.

(c) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ span a subspace V, and V contains some linearly independent list of three vectors, then $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a basis of V.

(d) If $U = \operatorname{span}\{\mathbf{u}_1, \mathbf{u}_2\}$ lies inside a plane P through the origin in $\mathbb{R}^3$, then $U = P$.

(e) If V is a subspace of $\mathbb{R}^6$ with $\dim V = 6$, then $V = \mathbb{R}^6$.

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6. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be a basis of a subspace V of $\mathbb{R}^n$, and let

$$\mathbf{w}_1 = \mathbf{v}_1, \qquad \mathbf{w}_2 = \mathbf{v}_1 + \mathbf{v}_2, \qquad \mathbf{w}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3.$$

(a) Prove that $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ is a basis of V. Check only that the list lies in V and is independent, and say why it also spans V.

(b) Find the coordinates of $\mathbf{v}_3$, and of $\mathbf{v}_1 + 2\mathbf{v}_2 + 3\mathbf{v}_3$, relative to $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$.

Statements	Reasons

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