Linear Algebra Unit 1 worksheet: Linear combination

Name _____ Date _____ Period _____

1. Let $\mathbf{u} = (-4, 3, 6, 6, -5)$, $\mathbf{v} = (-2, -5, 1, 6, 1)$ and $\mathbf{w} = (1, 4, 0, 6, -3)$ in $\mathbb{R}^5$. Find the linear combination $2\mathbf{u} - 3\mathbf{v} + 2\mathbf{w}$.

2. Let $\mathbf{v}_1 = (-2, 5, -3, 0)$ and $\mathbf{v}_2 = (-3, 3, 3, 3)$ in $\mathbb{R}^4$. Show that $\mathbf{b} = (-2, 14, -18, -6)$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, and find the weights.

3. Let $\mathbf{f}_1 = (1, 0, 0, 0)$, $\mathbf{f}_2 = (1, 1, 0, 0)$, $\mathbf{f}_3 = (1, 1, 1, 0)$ and $\mathbf{f}_4 = (1, 1, 1, 1)$ in $\mathbb{R}^4$.

(a) Find the weights that write any $\mathbf{x} = (x_1, x_2, x_3, x_4)$ as $c_1\mathbf{f}_1 + c_2\mathbf{f}_2 + c_3\mathbf{f}_3 + c_4\mathbf{f}_4$, and explain why no other weights work.

(b) Write (5, −2, 0, 3) as a combination of $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4$.

(c) Which vectors $\mathbf{x}$ can be written with all four weights positive?

4. Vectors $\mathbf{u}$ and $\mathbf{v}$ in some $\mathbb{R}^n$ are used to build $\mathbf{x} = \mathbf{u} + 2\mathbf{v}$ and $\mathbf{y} = 3\mathbf{u} - \mathbf{v}$.

(a) Write $\mathbf{u}$ and $\mathbf{v}$ as linear combinations of $\mathbf{x}$ and $\mathbf{y}$.

(b) Write $4\mathbf{u} + \mathbf{v}$ as a linear combination of $\mathbf{x}$ and $\mathbf{y}$. Check your answer by writing it back in terms of $\mathbf{u}$ and $\mathbf{v}$.

5. Let $\mathbf{v}_1 = (1, 0, 2, -1)$, $\mathbf{v}_2 = (0, 1, 1, 3)$, $\mathbf{v}_3 = (2, -1, 3, -5)$ and $\mathbf{b} = (3, 2, 8, 3)$.

(a) Find every list of weights c_1, c_2, c_3 with $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$.

(b) Which list in (a) has $c_1 = 0$? Use it to write $\mathbf{b}$ as a combination of $\mathbf{v}_2$ and $\mathbf{v}_3$ alone.

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6. Mei's embroidery machine moves its needle by whole stitches of two kinds, $\mathbf{a} = (3, 2)$ and $\mathbf{b} = (1, 3)$, in millimeters (across, up). It can sew either stitch forward or backward (a backward stitch moves by $-\mathbf{a}$ or $-\mathbf{b}$), but never part of a stitch. The needle starts at the origin, and the order of the stitches does not matter.

(a) For each of the points (5, 8), (5, 1) and (4, 3), decide whether the needle can land on it, and if so say which stitches to sew.

(b) Show that the needle can land on a point (x, y) with whole-number coordinates exactly when $3x - y$ is a multiple of 7.

7. Let $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{b}$ be vectors in $\mathbb{R}^n$ with $\mathbf{b} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

(a) Prove that if $c_1 \neq 0$, then $\mathbf{v}_1$ is a linear combination of $\mathbf{b}$ and $\mathbf{v}_2$.

(b) Give actual vectors in $\mathbb{R}^2$ showing that when $c_1 = 0$, $\mathbf{v}_1$ need not be a linear combination of $\mathbf{b}$ and $\mathbf{v}_2$.

Statements	Reasons

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