Linear Algebra Unit 1 worksheet: Linear independence

Name _____ Date _____ Period _____

1. Each list below is linearly dependent. For each one, give a dependence relation with integer weights, and say which vectors of the list are linear combinations of the others. Do not set up a system.

(a) $\mathbf{a} = (2, -1, 3)$, $\mathbf{b} = (-6, 3, -9)$, $\mathbf{c} = (1, 1, 1)$

(b) $\mathbf{p} = (1, 0, 2, -1)$, $\mathbf{q} = (0, 0, 0, 0)$, $\mathbf{r} = (3, 1, 1, 1)$

(c) $\mathbf{x} = (2, -1)$, $\mathbf{y} = (1, 3)$, $\mathbf{z} = (5, 1)$

(d) $\mathbf{d}_1 = (1, 1, 0, 0)$, $\mathbf{d}_2 = (0, 1, 1, 0)$, $\mathbf{d}_3 = (1, 2, 1, 0)$, $\mathbf{d}_4 = (0, 0, 0, 1)$

2. Vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{b}$ in some $\mathbb{R}^n$ satisfy both

$$\mathbf{b} = 2\mathbf{v}_1 - \mathbf{v}_2 + 3\mathbf{v}_3 \quad \text{and} \quad \mathbf{b} = \mathbf{v}_1 + \mathbf{v}_2 + 4\mathbf{v}_3.$$

(a) Find a dependence relation among $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

(b) Write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2$ and $\mathbf{v}_3$.

(c) Write $\mathbf{b}$ as a linear combination of $\mathbf{v}_2$ and $\mathbf{v}_3$ alone.

(d) Explain why $\mathbf{b}$ has infinitely many lists of weights as a combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and give infinitely many such lists in terms of one number t.

3. Let $\mathbf{v}_1 = (1, 2, 0, -1)$, $\mathbf{v}_2 = (2, 3, 1, 0)$, $\mathbf{v}_3 = (0, 1, -1, -2)$, $\mathbf{v}_4 = (0, 1, -1, 3)$ and $\mathbf{v}_5 = (3, 4, 2, -4)$ in $\mathbb{R}^4$.

(a) Row reduce and find every dependence relation $c_1\mathbf{v}_1 + \cdots + c_5\mathbf{v}_5 = \mathbf{0}$.

(b) Find the dependence relation with $c_1 = 0$ and $c_5 = 2$.

(c) Which of the five vectors are linear combinations of the other four? Explain.

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4. Show by row reduction that $\mathbf{v}_1 = (1, -2, -3, 1, -3)$, $\mathbf{v}_2 = (1, 0, 1, 3, 3)$ and $\mathbf{v}_3 = (6, -4, -2, 14, 6)$ in $\mathbb{R}^5$ are linearly dependent. Find every dependence relation among them, and use one to write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2$ and $\mathbf{v}_3$.

5. Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be a linearly independent list in $\mathbb{R}^n$, and let $\mathbf{w}$ be a vector of $\mathbb{R}^n$ that is not a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_k$.

(a) Prove that the list $\mathbf{v}_1, \ldots, \mathbf{v}_k, \mathbf{w}$ is linearly independent.

(b) Give actual vectors in $\mathbb{R}^2$ showing that the conclusion can fail if $\mathbf{w}$ is a linear combination of the others.

Statements	Reasons

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