Linear Algebra Unit 1 worksheet: Span

Name _____ Date _____ Period _____

1. Describe each span as the origin alone, a line through the origin, a plane through the origin, or all of $\mathbb{R}^2$ or $\mathbb{R}^3$. Give a reason for each.

(a) $\text{span}\{(2,5),(-1,3)\}$

(b) $\text{span}\{(-2,4,-6),(3,-6,9)\}$

(c) $\text{span}\{(1,0,2),(0,3,0),(2,3,4)\}$

(d) $\text{span}\{(1,1,0),(0,1,1),(1,0,1)\}$

(e) $\text{span}\{(3,-1,2,0)\}$

(f) $\text{span}\{(1,2),(2,4),(-3,-6)\}$

2. Let $\mathbf{v}_1 = (1,-2,-3)$ and $\mathbf{v}_2 = (-2,5,8)$ in $\mathbb{R}^3$.

(a) Row reduce with a general right side (x,y,z) to find one equation in x, y and z satisfied by exactly the vectors of $\text{span}\{\mathbf{v}_1,\mathbf{v}_2\}$.

(b) Use your equation to decide whether $\mathbf{b} = (-3,9,15)$ and $\mathbf{d} = (-1,9,15)$ are in the span. For each one that is, find the weights.

3. For a number h, let $\mathbf{v}_1 = (1,h,2)$ and $\mathbf{v}_2 = (h,4,2h)$ in $\mathbb{R}^3$.

(a) For which values of h is $\text{span}\{\mathbf{v}_1,\mathbf{v}_2\}$ a line? What is it for every other h?

(b) Show that for every h not found in (a) the span is one and the same set, and give its equation.

(c) For each h found in (a), give two equations describing the line.

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4. Let $\mathbf{v}_1 = (1, 1, 0, 1)$, $\mathbf{v}_2 = (-2, 0, 1, 2)$, $\mathbf{v}_3 = (0, 2, 1, -1)$ and $\mathbf{v}_4 = (3, 1, -1, 2)$ in $\mathbb{R}^4$, and let $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$ be the standard vectors of $\mathbb{R}^4$.

(a) Do $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ span $\mathbb{R}^4$?

(b) Find one equation in x_1, x_2, x_3, x_4 satisfied by exactly the vectors of $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

(c) Which of $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$ are in this span?

(d) For which j do the five vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{e}_j$ span $\mathbb{R}^4$? Justify with the pivot test.

5. Decide whether each statement about vectors in $\mathbb{R}^3$ is true or false. Give a reason for a true statement and a counterexample with actual vectors for a false one.

(a) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^3$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ span $\mathbb{R}^3$ for every $\mathbf{v}_4$.

(b) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^3$, then $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ is a plane.

(c) If $\mathbf{b}$ is not in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ and not in $\text{span}\{\mathbf{v}_1, \mathbf{v}_3\}$, then $\mathbf{b}$ is not in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

6. Priya's office has three rooms on one air system with two dampers. Moving damper A by one notch from its usual setting changes the temperatures of (room 1, room 2, room 3) by $\mathbf{a} = (2, 1, -1)$ degrees Celsius; moving damper B by one notch changes them by $\mathbf{b} = (1, 2, 1)$. In this made-up model a damper can move any real number of notches (a negative number moves it the other way), and the changes add: settings c_1 and c_2 change the temperatures by $c_1\mathbf{a} + c_2\mathbf{b}$.

(a) Find one equation satisfied by exactly the changes (x, y, z) the two dampers can make.

(b) Can the dampers change the temperatures by (3, 1, −2)? By (1, 1, 1)? Where they can, give the settings.

(c) Describe every change the dampers can make that leaves room 3 unchanged, and the settings that make it.

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