Linear Algebra Unit 1 worksheet: Subspaces

Name _____ Date _____ Period _____

1. Decide whether each set is a subspace. For a yes, give the reason; for a no, show one condition failing with specific vectors.

(a) $\{(x, y, z) : x = 3z\}$ in $\mathbb{R}^3$

(b) $\{(x, y) : x \le y\}$ in $\mathbb{R}^2$

(c) $\{(x, y, z) : z = xy\}$ in $\mathbb{R}^3$

(d) $\{(x, y, z) : x = 0 \text{ or } y = 0\}$ in $\mathbb{R}^3$

(e) $\{(x, y, z) : 2x - y = 0 \text{ and } z = 5\}$ in $\mathbb{R}^3$

(f) $\{(x, y, z) : x^2 + y^2 + z^2 = 0\}$ in $\mathbb{R}^3$

2. A 2×2 table of numbers with rows (p, q) and (r, s) is recorded as the vector (p, q, r, s) in $\mathbb{R}^4$. Call a table balanced when its two row sums are equal and its two column sums are equal.

(a) Show from the three conditions, using general vectors, that the balanced tables form a subspace W of $\mathbb{R}^4$.

(b) Write W as a span.

(c) Do the tables whose two row sums and two column sums all equal 10 form a subspace? Explain.

3. Let

$$A = \begin{bmatrix} 1 & 2 & -2 & -2 & -3 \\ -1 & -1 & 4 & 2 & 5 \\ -2 & -3 & 6 & 5 & 10 \end{bmatrix}.$$

Write the solution set N of $A\mathbf{x} = \mathbf{0}$ as a span, and say why N is a subspace of $\mathbb{R}^5$.

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4. In $\mathbb{R}^4$, let $V = \text{span}\{(1,0,1,2),(0,1,-1,1)\}$ and let H be the set of vectors with $x_1 + 2x_2 - x_3 + x_4 = 0$.
(a) Explain why V and H are subspaces.

(b) Write the intersection $V \cap H$ (the vectors in both) as a span, and say why this shows $V \cap H$ is a subspace.

(c) Is V contained in H? Explain.

5. For real numbers s and t, let W_1 be the set of all vectors $(2s - t + 1,\ s + t + 2,\ 3s + 3,\ -t)$ and W_2 the set of all vectors $(2s - t + 3,\ s + t,\ 3s + 3,\ 1 - t)$ in $\mathbb{R}^4$. Each looks like a plane slid off the origin.
(a) Show that W_1 is not a subspace.

(b) Show that W_2 is a subspace by writing it as a span. (Hint: first find the s and t that give $\mathbf{0}$.)

(c) Decide whether (5, 4, 9, −1) is in W_2.

6. Let U and W be subspaces of $\mathbb{R}^n$, and let $U + W$ be the set of all sums $\mathbf{u} + \mathbf{w}$ with $\mathbf{u} \in U$ and $\mathbf{w} \in W$.
(a) Prove that $U + W$ is a subspace of $\mathbb{R}^n$.
(b) In $\mathbb{R}^3$, let $U = \text{span}\{(1,0,0)\}$ and $W = \text{span}\{(0,1,1)\}$. Write $U + W$ as a span and give an equation for it.

Statements	Reasons

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