# Linear Algebra Unit 1 worksheet: Adding and scaling vectors in n dimensions

Name _____ Date _____ Period _____

**1.** Let $\mathbf{u} = (3, -1, 0, 4, -2, 5)$, $\mathbf{v} = (-2, 6, 1, -3, 0, 4)$ and $\mathbf{w} = (1, 0, -5, 2, 7, -1)$ in $\mathbb{R}^6$. Compute each vector.

(a) $\mathbf{u} + \mathbf{v} + \mathbf{w}$

(b) $2\mathbf{v} - 3\mathbf{w}$

(c) $\mathbf{u} - (\mathbf{v} - \mathbf{w})$

(d) $\frac{1}{2}(\mathbf{u} + \mathbf{v}) - \mathbf{w}$

**2.** Solve $3\mathbf{x} + \mathbf{a} = \mathbf{b} - 2\mathbf{x}$ for $\mathbf{x}$ in $\mathbb{R}^4$, where $\mathbf{a} = (-1, -6, 7, 6)$ and $\mathbf{b} = (9, 19, 12, -9)$. Check your answer in the original equation.

**3.** Use the definition of equal vectors.

(a) Find all real numbers $x$ and $y$ with $(x^2, x + y, y^2 - 1, xy) = (4, 1, 8, -6)$.

(b) Find every real number $t$ for which $(t^2 - 1, t^2 + t, t^3 - t)$ is the zero vector of $\mathbb{R}^3$.

(c) Find every scalar $k$ with $k(k, 1, 2, -k) = (4, 2, 4, -4)$.

**4.** Find the vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^4$ with $\mathbf{x} + 2\mathbf{y} = \mathbf{a}$ and $\mathbf{x} - \mathbf{y} = \mathbf{b}$, where $\mathbf{a} = (10, 2, 5, 0)$ and $\mathbf{b} = (1, -7, -4, -6)$.

**5.** Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{x}$ in $\mathbb{R}^n$ satisfy

$$3(\mathbf{x} - 2\mathbf{u}) + 2\mathbf{v} = 5\mathbf{x} - (\mathbf{u} - 4\mathbf{v}).$$

(a) Solve for $\mathbf{x}$ in terms of $\mathbf{u}$ and $\mathbf{v}$, naming the properties you use.

(b) Find $\mathbf{x}$ when $\mathbf{u} = (2, -4, 0, 6)$ and $\mathbf{v} = (1, 3, -2, -5)$, and check it in the original equation.


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**6.** The points $\mathbf{p} = (7, -4, 1, -6)$, $\mathbf{q} = (4, 3, -1, 6)$ and $\mathbf{r} = (-2, 7, 3, 0)$ of $\mathbb{R}^4$ are the corners of a triangle.

(a) Find the midpoint $\mathbf{m}$ of the segment from $\mathbf{q}$ to $\mathbf{r}$.

(b) Find the point $\mathbf{g}$ two thirds of the way from $\mathbf{p}$ to $\mathbf{m}$.

(c) Show that for any three points $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ of any $\mathbb{R}^n$, the point two thirds of the way from $\mathbf{p}$ to the midpoint of $\mathbf{q}$ and $\mathbf{r}$ is $\frac{1}{3}(\mathbf{p} + \mathbf{q} + \mathbf{r})$, naming the properties you use. Use this to check (b).

(d) Without computing, explain why the point two thirds of the way from $\mathbf{q}$ to the midpoint of $\mathbf{p}$ and $\mathbf{r}$ is the same point $\mathbf{g}$.

**7.** Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$ and $c$ a scalar.

(a) Prove entry by entry that $(-c)\mathbf{u} = -(c\mathbf{u})$.

(b) Prove, using only the eight properties and no entries, that if $\mathbf{u} + \mathbf{v} = \mathbf{0}$, then $\mathbf{v} = -\mathbf{u}$. Name the property behind each step.

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