# Linear Algebra Unit 2 worksheet: Solving a system by row reduction: one, none or infinitely many solutions

Name _____ Date _____ Period _____

**1.** Each matrix is the result of row reducing the augmented matrix of a system. The variables are $x$, $y$, $z$, $w$ in (a) and $x$, $y$, $z$ in (b) and (c). Write the solution set with parameters, or say that there is no solution.

(a) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 4 & -2 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 7 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

**2.** Solve the system in $x$, $y$, $z$, $w$ whose augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & -4 & 7 \\ -3 & 7 & 2 & 11 & -22 \\ -2 & 1 & -5 & 9 & -10 \end{array}\right]$$

Reduce to reduced row echelon form and write the solution set with a parameter.

**3.** Consider the system

$$\begin{aligned} x + y + z &= 4 \\ 2x + 3y + z &= 11 \\ -x + y - 3z &= 2 \\ 3x + 2y + 4z &= 9 \end{aligned}$$

Solve it by row reduction. Say how many solutions it has, and write the solution set if there is one.


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**4.** The augmented matrix of a system in $x$, $y$, $z$ is

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 5 \\ 2 & 7 & -4 & 6 \\ -3 & -12 & 9 & c \end{array}\right]$$

where $c$ is a constant.

(a) Find the value of $c$ for which the system is consistent.

(b) For that value of $c$, write the solution set.

**5.** Arjun buys 20 postage stamps for 13 dollars. Some cost 50 cents, some 70 cents and some 90 cents. Let $a$, $b$ and $c$ be the numbers of 50-cent, 70-cent and 90-cent stamps.

(a) Write two equations, one for the count and one for the cost in cents. Row reduce the augmented matrix and write the solution set with a parameter.

(b) A number of stamps must be a whole number and cannot be negative. How many different purchases are possible? Give the one with the most 90-cent stamps.

**6.** In the system

$$\begin{aligned} kx + y + z &= 1 \\ x + ky + z &= 1 \\ x + y + kz &= 1 \end{aligned}$$

the letter $k$ appears once in each equation. Start with $R_1 \leftrightarrow R_3$, so that the first pivot is not an expression in $k$, and reduce with $k$ carried along. Then sort every real $k$ by the number of solutions, and write the solution set in each case that has solutions.


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