Linear Algebra Unit 4 worksheet: Determinants by cofactor expansion

Name _____ Date _____ Period _____

1. Let

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 5 & 4 \\ -3 & 0 & 2 \end{bmatrix}.$$

(a) Find $\det A$ by expanding along row 1. Find only the cofactors you need, and write each one down.

(b) Row 3 also has a zero. Find only the cofactors you need to expand along row 3, and check that you get the same $\det A$.

2. Find $\det M$ for the matrix below. Say which row or column you expand along first, and why.

$$M = \begin{bmatrix} 2 & 2 & 0 & -3 \\ -1 & -4 & 2 & 2 \\ 1 & 4 & 0 & -2 \\ -3 & 0 & 0 & 1 \end{bmatrix}$$

3. (a) Find $\det L$ for

$$L = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ 1 & -4 & \frac{1}{2} & 0 & 0 \\ 7 & 2 & 6 & -1 & 0 \\ -3 & 8 & 1 & 9 & 4 \end{bmatrix}$$

(b) Find $\det B$ for

$$B = \begin{bmatrix} 2 & 1 & 4 & -1 \\ 3 & 1 & 0 & 5 \\ 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

(c) Multiply the diagonal entries of B. Why does that product not give $\det B$, and which entry is to blame?

4. For a number k, let

$$K = \begin{bmatrix} 1 & k & 0 & 2 \\ 0 & 0 & 3 & 1 \\ 2 & 0 & 1 & k \\ 1 & 1 & 3 & 3 \end{bmatrix}.$$

(a) Find $\det K$ as a polynomial in k, and factor it.

(b) For which k does the system $K\mathbf{x} = \mathbf{b}$ fail to have exactly one solution?

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5. Find $\det Q$ for

$$Q = \begin{bmatrix} 3 & 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 4 & 1 \\ 0 & 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 3 & 1 \\ 0 & 1 & -1 & 0 & 2 \end{bmatrix}$$

No row or column has only one nonzero entry, so choose a line with two, and choose afresh inside each minor.

6. Full cofactor expansion of an $n \times n$ matrix with no zero entries forms $n!$ products, while row reduction to triangular form needs about $\frac{1}{3}n^3$ multiplications. A computer forms 10^9 products per second.

(a) Find the smallest $n \geq 2$ for which $n! > \frac{1}{3}n^3$.

(b) What is the largest n for which full expansion on this computer finishes within one day (86,400 seconds)?

7. For $n \geq 1$, let D_n be the determinant of the $n \times n$ matrix with 3 in every diagonal position, 1 in every position just above or just below the diagonal, and 0 everywhere else. For example,

$$D_4 = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}.$$

(a) Find D_1 and D_2.

(b) For $n \geq 3$, expand along row 1, then expand the second minor down its column 1, to show that $D_n = 3D_{n-1} - D_{n-2}$.

(c) Find D_6.

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