Linear Algebra Unit 5 worksheet: Finding an eigenvector

Name _____ Date _____ Period _____

1. The matrix

$$A = \begin{bmatrix} 8 & 1 \\ -6 & 3 \end{bmatrix}$$

has eigenvalues 5 and 6. Find a basis for each eigenspace, and check each basis vector by one multiplication.

2. The matrix

$$A = \begin{bmatrix} 10 & -6 & -6 \\ -6 & 10 & 6 \\ 9 & -9 & -5 \end{bmatrix}$$

has eigenvalues 4, with algebraic multiplicity 2, and 7.

(a) Find a basis for each eigenspace.

(b) Give the geometric multiplicity of each eigenvalue, and decide whether $\mathbb{R}^3$ has a basis made of eigenvectors of A.

3. The matrix

$$A = \begin{bmatrix} -1 & -3 & 3 \\ -4 & -1 & 4 \\ -4 & -3 & 6 \end{bmatrix}$$

has eigenvalues 2, −1 and 3.

(a) Find a basis for each eigenspace.

(b) Explain why the three vectors found in (a) together form a basis of $\mathbb{R}^3$.

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4. Let

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 6 & -6 & 2 \end{bmatrix}.$$

(a) Five vectors are offered: $(0, 0, 0)$, $(2, 2, 0)$, $(1, 2, 2)$, $(0, 1, 2)$ and $(-1, -1, -1)$. Multiply to test them: which are eigenvectors of A, and for which eigenvalues?

(b) Without the characteristic polynomial, list every eigenvalue of A and give a basis for each eigenspace. Explain why nothing is missing.

5. Use the geometry first, then the matrix.

(a) The reflection of $\mathbb{R}^2$ across the line $y = 3x$ has matrix

$$R = \frac{1}{5}\begin{bmatrix} -4 & 3 \\ 3 & 4 \end{bmatrix}.$$

Which vectors does the reflection leave unchanged, and which does it reverse? Name the eigenvalues and eigenspaces this predicts, then confirm them by row reducing $R - I$ and $R + I$.

(b) The quarter turn of $\mathbb{R}^3$ about the x_3-axis (counterclockwise, seen from above) has matrix

$$Q = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find E_1 by row reduction and describe it geometrically. Then decide whether -1 is an eigenvalue of Q, and say what that means for the turn.

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6. Both matrices below are upper triangular with characteristic polynomial $(3-\lambda)^2(1-\lambda)$:

$$F = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) For each matrix, find a basis for E_3 and for E_1, and give the geometric multiplicity of each eigenvalue.

(b) Which of the two matrices has a basis of $\mathbb{R}^3$ made of its eigenvectors? Give one such basis, and explain why the other matrix has none.

7. Let $\mathbf{v}$ be an eigenvector of the $n \times n$ matrix A for the eigenvalue λ.

(a) Suppose $\mathbf{v}$ is also an eigenvector of a second $n \times n$ matrix B, for the eigenvalue μ. Prove that $\mathbf{v}$ is an eigenvector of AB and of $A + B$, and name the eigenvalue in each case.

(b) Suppose A is invertible. Prove that $\lambda \neq 0$ and that $\mathbf{v}$ is an eigenvector of A^{-1} for $\dfrac{1}{\lambda}$.

Statements	Reasons

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