Perfect, abundant, deficient and amicable numbers

Add up the factors of a number, leaving out the number itself. Sometimes you get the number back exactly. What can the sum tell you?

WORDS TO KNOW

Proper factors: all the factors of a number except the number itself. The proper factors of 10 are 1, 2 and 5.

Perfect: the proper factors add to exactly the number. 6 is perfect: 1 + 2 + 3 = 6.

Deficient: the proper factors add to less than the number.

$s(n)$: the sum of the proper factors of n. So $s(10) = 1 + 2 + 5 = 8$.

Abundant: the proper factors add to more than the number.

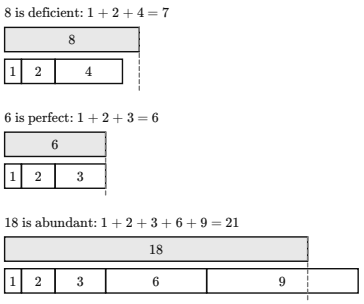
Amicable pair: two different numbers where the proper factors of each one add to the other.

KEY IDEAS

1. Lay the factors end to end

Every whole number from 1 up is exactly one of perfect, abundant or deficient, because the sum is equal to, more than or less than the number.

To find $s(n)$, list the factors in pairs that multiply to n, so none is missed. Then add all of them except n.



2. Some numbers you can sort at once

A prime p has only 1 as a proper factor, so $s(p) = 1$. Every prime is deficient.

A power of 2 falls short by exactly one: $s(8) = 1 + 2 + 4 = 7$.

A multiple of a perfect or abundant number (other than the number itself) is abundant.

Numbers with many small factors tend to be abundant. Numbers with few factors tend to be deficient. Perfect numbers, like 6, 28 and 496, are rare.

3. Friendly pairs

a and b are an amicable pair when $s(a) = b$ **and** $s(b) = a$, and a is not b.

The smallest amicable pair is 220 and 284: $s(220) = 284$ and $s(284) = 220$.

WATCH OUT

- **Leave the number itself out.** The factors of 14 are 1, 2, 7 and 14, but $s(14) = 1 + 2 + 7 = 10$, not 24.
- **Check both directions.** $s(a) = b$ alone does not make a pair. You must also check that $s(b) = a$.
- **Do not miss a factor.** Pair each factor with its partner, and stop when the next number to try, times itself, is more than n.

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WORKED EXAMPLES

Example 1. Is 42 perfect, abundant or deficient? Is 35?

1. 42 = 1 × 42 = 2 × 21 = 3 × 14 = 6 × 7. The numbers 4 and 5 do not divide 42, and 7 × 7 = 49 is more than 42, so the list is complete.
2. $s(42) = 1 + 2 + 3 + 6 + 7 + 14 + 21 = 54$, and 54 is more than 42.
3. 35 = 1 × 35 = 5 × 7. The numbers 2, 3 and 4 do not divide 35, and 6 × 6 = 36 is more than 35. So $s(35) = 1 + 5 + 7 = 13$, less than 35.

Answer: 42 is abundant and 35 is deficient. (We could also see 42 is abundant because 42 = 7 × 6, a multiple of the perfect number 6.)

Example 2. Use $75 = 3 \times 5^2$ to find $s(75)$.

	1	5	25
1	1	5	25
3	3	15	75

1. Every factor of 75 is 1 or 3, times 1, 5 or 25. The grid lists them all.
2. Leave out 75 and add the rest: 1 + 5 + 25 + 3 + 15 = 49.

Answer: $s(75) = 49$, so 75 is deficient.

Example 3. Mo finds $s(12) = 16$ and says 12 and 16 are an amicable pair. Is he right?

1. The proper factors of 12 are 1, 2, 3, 4 and 6. They add to 16, so that half is true.
2. Now check the other way. The proper factors of 16 are 1, 2, 4 and 8, so $s(16) = 15$.
3. 15 is not 12, so the sum from 16 does not lead back to 12.

Answer: no. $s(12) = 16$, but $s(16) = 15$, not 12.

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