Tests for 3 and 9

Can you tell whether a big number splits evenly into threes or nines without doing the division?

WORDS TO KNOW

Divisible by 9: the number is 9 times a whole number, so it splits into groups of 9 with nothing left over. Divisible by 3 means the same with groups of 3.

Remainder: what is left over after you make as many full groups as you can.

Digit sum: what you get when you add the digits of a number. The digit sum of 5382 is 5 + 3 + 8 + 2 = 18.

KEY IDEAS

1. Add the digits

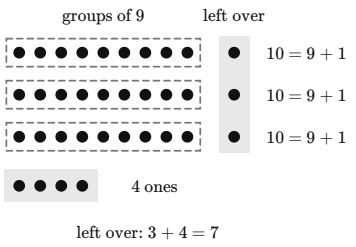
For 3 and 9 the last digit is no help: 13 and 33 both end in 3, but only 33 is a multiple of 3. Every digit matters, so the test uses all of them.

> A whole number is divisible by 9 exactly when its digit sum is divisible by 9.
>
> A whole number is divisible by 3 exactly when its digit sum is divisible by 3.

If the digit sum is still big, add its digits again. The test works on it too.

2. Why it works: every ten is a nine and one more

10 = 9 + 1, 100 = 99 + 1, 1000 = 999 + 1, and 9, 99, 999 are all multiples of 9. So each ten, hundred or thousand makes full groups of 9 with exactly one left over. In 34, the three tens leave 3 over and the four ones leave 4 over: 3 + 4 = 7, the digit sum.



> A whole number leaves the same remainder as its digit sum, when you divide by 9 and when you divide by 3.

3. Nine is three threes

A group of 9 is three groups of 3. So a number divisible by 9 is always divisible by 3. The reverse is not true: a number can pass the test for 3 and fail the test for 9.

WATCH OUT

- **The last digit does not decide.** 23 ends in 3, but 23 = 3 × 7 + 2. Add all the digits.
- **The test is only for 3 and 9.** It does not work for 6: 75 has digit sum 12, a multiple of 6, but 75 is odd, so it is not divisible by 6. It does not work for 27 either.
- **Passing for 3 is not passing for 9.** A digit sum of 12 is divisible by 3 but not by 9.

WORKED EXAMPLES

Example 1. Is 738 divisible by 3? By 9?

1. Add the digits: 7 + 3 + 8 = 18.
2. 18 = 9 × 2, so 18 is divisible by 9, and so by 3 as well.

Answer: 738 is divisible by both 3 and 9. **Check:** 9 × 82 = 738.

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Example 2. Rosa has 853 beads. How many are left over in groups of 9? In groups of 3?

1. Add the digits: 8 + 5 + 3 = 16.
2. Groups of 9: 16 = 9 + 7, so 16 leaves 7. The number leaves the same remainder as its digit sum.
3. Groups of 3: 16 = 3 × 5 + 1, so 1 is left.

Answer: 7 beads are left over in groups of 9, and 1 bead in groups of 3. **Check:** 9 × 94 = 846 and 853 − 846 = 7.

Example 3. The number $1d5$ has a missing middle digit d. Which digits make it divisible by 9? By 3?

1. The digit sum is $1 + d + 5 = 6 + d$. A digit is 0 to 9, so $6 + d$ is from 6 to 15.
2. For 9: the only multiple of 9 from 6 to 15 is 9. So $6 + d = 9$ and $d = 3$.
3. For 3: the multiples of 3 from 6 to 15 are 6, 9, 12 and 15. So d is 0, 3, 6 or 9.

Answer: by 9, only 135. By 3: 105, 135, 165 and 195. **Check:** 135 = 9 × 15.

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