Prime factorization: factor trees and the ladder

Every whole number greater than 1 is built from primes. How do you find the primes a number is made of?

WORDS TO KNOW

Factor pair: two whole numbers that multiply to the number. 4 and 6 are a factor pair of 24.

Prime factorization: the number written as a product of primes only. For 24 it is $2 \cdot 2 \cdot 2 \cdot 3$.

Factor tree: a diagram that splits a number into factor pairs until every branch ends in a prime.

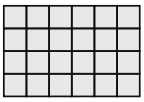
Prime: a whole number greater than 1 whose only factors are 1 and itself, such as 2, 3, 5, 7, 11.

Exponent: a small raised number that counts how many times a prime repeats. 2^3 means $2 \cdot 2 \cdot 2$, so $24 = 2^3 \cdot 3$.

Ladder: a list of divisions by primes, from the number down to 1.

KEY IDEAS

1. A factor tree splits until only primes are left



$24 = 4 \cdot 6$



$7 = 1 \cdot 7$

Write the number at the top. Split it into any factor pair without 1. Circle each prime. Split every number that is not prime again.

> The circled primes multiply back to the top number.
>
> Write them smallest first, with exponents for repeats. An exponent of 1 is not written.

Any first split works. Different trees for the same number always end in the same primes, each the same number of times.

2. The ladder divides by the smallest prime that goes in

> Try 2, then 3, then 5, then 7, and so on. Divide by the first prime that goes in.
>
> Try the same prime again before you move on. Stop when you reach 1.
>
> The primes down the left side are the prime factorization, already in order.

To decide whether a number is prime, you only need to try the primes p where $p \times p$ is no bigger than the number.

3. Check your answer two ways

> Every number in it is prime, and the product is the number you started with.

WATCH OUT

- **Never use 1.** 1 is not prime, and the split $1 \cdot 20$ gives back the same 20. Every split must use two numbers bigger than 1.
- **Divide by primes only.** On a ladder, dividing by 4 or 9 puts a number that is not prime on the left. Keep using 2 until it no longer goes in.
- **An exponent is not multiplying.** 3^4 means $3 \cdot 3 \cdot 3 \cdot 3 = 81$, not $3 \cdot 4 = 12$.

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WORKED EXAMPLES

Example 1. Find the prime factorization of 220 with a factor tree.

1. 220 ends in 0, so 10 is a factor: 220 = 10 · 22.
2. Neither is prime. Split again: 10 = 2 · 5 and 22 = 2 · 11.
3. The circled primes are 2, 5, 2 and 11. Smallest first: 2 · 2 · 5 · 11.

Answer: $220 = 2^2 \cdot 5 \cdot 11$. **Check:** 4 · 5 · 11 = 20 · 11 = 220.

Example 2. Find the prime factorization of 594 with the ladder.

Divide by	Number
2	594
3	297
3	99
3	33
11	11
	1

1. 594 is even: 594 ÷ 2 = 297. Now 297 is odd, so move on to 3.
2. The digits of 297 add to 18, so 3 goes in: 297 ÷ 3 = 99. Keep trying 3: 99 ÷ 3 = 33 and 33 ÷ 3 = 11.
3. 11 is prime: 11 ÷ 11 = 1. Stop.

Answer: $594 = 2 \cdot 3^3 \cdot 11$. **Check:** 2 · 27 · 11 = 54 · 11 = 594.

Example 3. Can 1,225 tiles make a square? Use the prime factorization.

1. 1,225 is odd, and its digits add to 10, so 2 and 3 do not go in. It ends in 5: 1,225 ÷ 5 = 245, then 245 ÷ 5 = 49.
2. 49 does not end in 0 or 5, so try 7: 49 ÷ 7 = 7 and 7 ÷ 7 = 1. So $1{,}225 = 5^2 \cdot 7^2$.
3. Each prime comes twice, so the primes share out into two equal groups: (5 · 7) · (5 · 7) = 35 · 35.

Answer: yes, a square 35 tiles on each side. **Check:** 35 · 35 = 1,225.

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