Greatest common factor by listing factors

You want to split two amounts into identical shares with nothing left over, and make as many shares as you can. How many shares is that?

WORDS TO KNOW

Factor: a whole number that divides another with nothing left over. 4 is a factor of 12.

Common factor: a number that is a factor of both numbers.

Relatively prime: two numbers whose GCF is 1, such as 8 and 15.

Factor pair: two factors that multiply to the number, such as $3 \times 4 = 12$.

Greatest common factor (GCF): the largest common factor. GCF(26, 39) = 13.

KEY IDEAS

1. List the factors in pairs

Try 1, 2, 3, and so on. Each time a number divides, its partner divides too, so you get two factors at once.

> Stop when the number you are about to try, times itself, is more than the number. Then the list is complete.

For 20: 1×20, 2×10, 4×5. You can stop at 5, since $5 \times 5 = 25$ is more than 20. The factors are 1, 2, 4, 5, 10, 20.

2. The GCF is the biggest number on both lists

> Write both factor lists. The common factors are the numbers on both. The GCF is the largest of them.

Every common factor is a factor of the GCF. That gives a quick check on your lists.

3. A shortcut: work down the smaller number's list

The GCF is never bigger than the smaller number. So list only the factors of the smaller number.

> Test them from the top. The first one that also divides the larger number is the GCF.

4. Reading a story

> **Most identical parcels**, each holding some of every item: the number of parcels is the GCF.
>
> **Biggest equal groups**, each holding one kind of item: the size of a group is the GCF.

Divide each amount by the GCF to get the other number the story asks for.

WATCH OUT

- **Do not drop a partner.** If 3 is a factor of 24, so is 8. A missing factor could be the GCF itself.
- **A common factor is not always the greatest.** For 40 and 60, 10 is a common factor, but the GCF is 20.
- **Read what the story counts.** The GCF may be the number of parcels or the size of each group.

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WORKED EXAMPLES

Example 1. List the common factors of 45 and 72, and find their GCF.

1. Factors of 45: 1 × 45, 3 × 15, 5 × 9. Stop, since 7 × 7 = 49 is more than 45.
2. Factors of 72: 1 × 72, 2 × 36, 3 × 24, 4 × 18, 6 × 12, 8 × 9. Stop, since 9 × 9 = 81 is more than 72.

Number	Factors
45	1, 3, 5, 9, 15, 45
72	1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Answer: the common factors are 1, 3 and 9, so GCF(45, 72) = 9. **Check:** 45 = 9 × 5 and 72 = 9 × 8.

Example 2. Find GCF(64, 80) by working down the smaller list.

1. The factors of 64 are 1, 2, 4, 8, 16, 32, 64.
2. Test from the top. 64 does not divide 80. 80 ÷ 32 is 2 remainder 16, so 32 fails.
3. 80 ÷ 16 = 5 exactly. So 16 is the first one that works.

Answer: GCF(64, 80) = 16. **Check:** 64 = 16 × 4 and 80 = 16 × 5.

Example 3. Kim has 70 apples and 84 oranges. What is the largest number of identical bags she can fill with nothing left over? What goes in each bag?

1. Each bag gets some of both fruits, so the number of bags must divide 70 and 84. The most bags is the GCF.
2. Work down the factors of 70: 70, 35, 14, 10, 7, 5, 2, 1. 70 does not divide 84, and 84 ÷ 35 is 2 remainder 14.
3. 84 ÷ 14 = 6 exactly. So GCF(70, 84) = 14.
4. Each bag gets 70 ÷ 14 = 5 apples and 84 ÷ 14 = 6 oranges.

Answer: 14 bags, each with 5 apples and 6 oranges. **Check:** 14 × 5 = 70 and 14 × 6 = 84.

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