Every number has one prime factorization

Two people split the same number into primes, starting in different ways. Do they always end with the same primes?

WORDS TO KNOW

Prime factorization: a number written as a product of primes only, such as $140 = 2 \cdot 2 \cdot 5 \cdot 7$.

The same factorization: the same primes, each the same number of times, in any order. $2 \cdot 5 \cdot 2 \cdot 7$ and $7 \cdot 2 \cdot 2 \cdot 5$ are the same.

Perfect square: a whole number times itself, such as $36 = 6 \cdot 6$.

Tidy form: the primes listed smallest first, with repeats written as powers: $140 = 2^2 \cdot 5 \cdot 7$.

Divides: goes into a number with nothing left over. 7 divides 14.

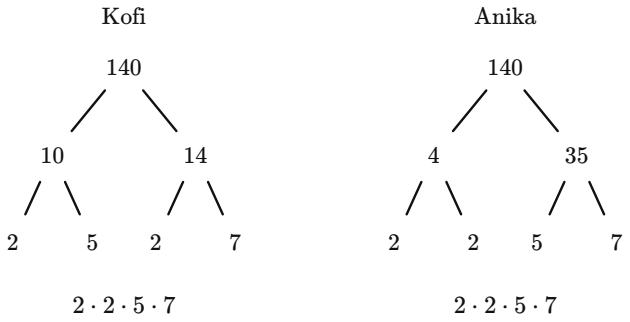
KEY IDEAS

1. Only one way to split into primes

Kofi starts 140 as $10 \cdot 14$. Anika starts it as $4 \cdot 35$. Both end with the same primes: two 2s, one 5 and one 7.

> Every whole number greater than 1 can be written as a product of primes in exactly one way, apart from the order.

The reason is a fact about primes: if a prime divides a product, it divides at least one of the numbers being multiplied.



2. Read the divisors from the primes

> A number d divides n exactly when every prime of d shows up in n at least as many times as in d.

So $28 = 2^2 \cdot 7$ divides $140 = 2^2 \cdot 5 \cdot 7$. But $8 = 2^3$ needs three 2s, and 140 has only two, so 8 does not divide 140.

3. Squares have even powers

A square $m \cdot m$ uses each prime of m twice, so every power doubles.

> A whole number greater than 1 is a perfect square exactly when every power in its prime factorization is even.

4. Different primes mean different numbers

If two prime factorizations differ in any prime, or in how many times a prime appears, they are different numbers. You can tell without multiplying.

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WATCH OUT

- **Only primes count.** 6 · 13 and 2 · 39 both make 78, but neither is a prime factorization. Keep splitting: both end as 2 · 3 · 13.
- **The divide rule is for primes only.** 10 divides 4 · 15 = 60, but 10 divides neither 4 nor 15.
- **Every repeat counts.** 2 · 5 · 7 and 2 · 2 · 5 · 7 are different factorizations, of 70 and of 140.

WORKED EXAMPLES

Example 1. Find the prime factorization of 456 starting from 12 · 38, and again from 6 · 76.

1. From 12 · 38: 12 = 2 · 2 · 3 and 38 = 2 · 19. The primes are three 2s, one 3 and one 19.
2. From 6 · 76: 6 = 2 · 3 and 76 = 2 · 2 · 19. Again three 2s, one 3 and one 19.

Answer: both give $456 = 2^3 \cdot 3 \cdot 19$. **Check:** 8 · 3 · 19 = 24 · 19 = 456.

Example 2. Let $n = 2^2 \cdot 5 \cdot 7^2$. Without dividing, does 14 divide n? Does 25?

1. 14 = 2 · 7 needs one 2 and one 7. The number n has two 2s and two 7s, so 14 divides it.
2. Take one 2 and one 7 out. What is left is 2 · 5 · 7 = 70, so $n = 14 \cdot 70$.
3. $25 = 5^2$ needs two 5s. The number n has only one, and it has no other factorization. So 25 does not divide n.

Answer: 14 divides n, and 25 does not. **Check:** $n = 4 \cdot 5 \cdot 49 = 980$, and 14 · 70 = 980.

Example 3. Is $2^6 \cdot 3^2$ a perfect square? Is $2^2 \cdot 3 \cdot 7^2$?

1. In $2^6 \cdot 3^2$, the powers 6 and 2 are both even. Halve them: $2^3 \cdot 3 = 24$.
2. In $2^2 \cdot 3 \cdot 7^2$, the power of 3 is 1, which is odd.

Answer: $2^6 \cdot 3^2 = 576 = 24 \cdot 24$ is a perfect square. $2^2 \cdot 3 \cdot 7^2 = 588$ is not. **Check:** 24 · 24 = 576, and 588 lies between 24 · 24 = 576 and 25 · 25 = 625.

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