Math 6 Unit 1 quiz 1: Tests for three and nine, The sieve of Eratosthenes, Prime factorization (answer key)

8 questions, 21 points

ANSWERS AT A GLANCE

1. Both counts are multiples of 9, so the true total is too, but 3,815 is not.
2. (a) $259 = 7 \times 37$ (b) $341 = 11 \times 31$ (c) $323 = 17 \times 19$
3. $2 \cdot 3^3$ is larger, by 6 (54 against 48).
4. (a) take away 4 (b) add 2
5. (a) Every order has digit sum 18, which is divisible by 9. (b) No: every order has digit sum 17, which is not divisible by 3.
6. (a) The multiples of 3 take turns being odd and even, and the even ones went out in round 2, so only every other multiple of 3 is still lit, and those are 6 apart. (b) 16 (c) 57 and 75
7. (a) $A = 1{,}610$, $B = 5$, $C = 161$, $D = 23$ (b) $1{,}610 = 2 \cdot 5 \cdot 7 \cdot 23$
8. (a) Round 3; for example $9 = 3 \times 3$ is still lit. (b) 9, 21, 27, 33, 39, 51, 57, 63, 69 (c) Only round 7, which puts out 49.

WORKED SOLUTIONS

1. Lily's class folds paper cranes for a display. The first team has 1,368 cranes and the second team has 2,457. Jin adds the two counts and gets a total of 3,815. Use digit sums to show that Jin's total is wrong, without adding again. (2 points)

ANSWER

Both counts are multiples of 9, so the true total is too, but 3,815 is not.

SOLUTION

The digit sum of 1,368 is 1 + 3 + 6 + 8 = 18, and the digit sum of 2,457 is 2 + 4 + 5 + 7 = 18. Both are divisible by 9, so each team's cranes make whole groups of 9. Putting the two teams' groups together still gives whole groups of 9, so the true total must be divisible by 9.

The digit sum of 3,815 is 3 + 8 + 1 + 5 = 17, which is not divisible by 9. So 3,815 cannot be the total. (The true total is 3,825, with digit sum 18.)

SCORING

1 point: both counts shown to be divisible by 9 by their digit sums, with the reason the total must then be divisible by 9. 1 point: the digit sum 17 of Jin's total, so it is not divisible by 9.

2. Each number is composite. Find its smallest prime factor, and write the number as that prime times another whole number. (3 points)

(a) 259 (b) 341 (c) 323

ANSWER

(a) $259 = 7 \times 37$ (b) $341 = 11 \times 31$ (c) $323 = 17 \times 19$

SOLUTION

Try the primes in order; the first one that divides is the smallest prime factor.

(a) 259 is odd, its digit sum 16 is not a multiple of 3, and it does not end in 0 or 5. $7 \times 37 = 259$. Smallest prime factor 7.

(b) 341 is odd, its digit sum 8 is not a multiple of 3, it does not end in 0 or 5, and $7 \times 48 = 336$ leaves remainder 5. $11 \times 31 = 341$. Smallest prime factor 11.

(c) 323 is odd, its digit sum 8 is not a multiple of 3, it does not end in 0 or 5; $7 \times 46 = 322$ leaves remainder 1, $11 \times 29 = 319$ leaves remainder 4, $13 \times 24 = 312$ leaves remainder 11. $17 \times 19 = 323$. Smallest prime factor 17.

SCORING

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1 point for each number written as its smallest prime factor times another whole number (7×37, 11×31, 17×19). A correct product whose first factor is not the smallest prime factor (for example 19×17 given as "smallest 19") earns no point for that part.

3. Which is larger, $2^4 \cdot 3$ or $2 \cdot 3^3$? By how much? (2 points)

ANSWER
$2 \cdot 3^3$ is larger, by 6 (54 against 48).

SOLUTION
$2^4 \cdot 3 = 16 \cdot 3 = 48$ and $2 \cdot 3^3 = 2 \cdot 27 = 54$. So $2 \cdot 3^3$ is larger, by $54 - 48 = 6$.

4. Naomi has 87,835 oranges at a citrus packing house and wants to pack all of them in bags, every bag the same size. Without dividing, find: (3 points)

(a) the fewest oranges Naomi must take away so that bags of 9 use every orange that is left;

(b) the fewest oranges Naomi must add so that bags of 3 use every orange.

ANSWER
(a) take away 4 (b) add 2

SOLUTION
The digit sum of 87,835 is 8 + 7 + 8 + 3 + 5 = 31. A number leaves the same remainder as its digit sum, on division by 9 and on division by 3.

(a) $31 = 9 \times 3 + 4$, so bags of 9 leave remainder 4. Taking that many away leaves a multiple of 9, and taking fewer still leaves some over.

(b) $31 = 3 \times 10 + 1$, so bags of 3 leave remainder 1. A full bag needs $3 - 1 = 2$ more.

SCORING
1 point: the digit sum 31. 1 point: (a) take away 4, the remainder on division by 9. 1 point: (b) add 2, from the remainder 1 on division by 3. A correct answer found by long division instead of the digit sum earns at most 2 points.

5. Ryan's padlock opens with a four-digit code that uses the digits 1, 3, 6 and 8, each once, but he has forgotten their order. (3 points)

(a) Explain why every code Ryan could make is divisible by 9.

(b) His sister says one of the digits is 7, not 8. If she is right, is any possible code divisible by 3? Explain.

ANSWER
(a) Every order has digit sum 18, which is divisible by 9. (b) No: every order has digit sum 17, which is not divisible by 3.

SOLUTION
(a) Whatever the order, the digit sum is 1 + 3 + 6 + 8 = 18, because adding in a different order gives the same sum. Since $18 = 9 \times 2$, every code is divisible by 9. For example $3{,}168 = 9 \times 352$.

(b) With the digits 1, 3, 6 and 7, every order has digit sum 1 + 3 + 6 + 7 = 17. Since $17 = 3 \times 5 + 2$, no code is divisible by 3: each one leaves remainder 2.

SCORING
(a) 1 point: the digit sum 18. 1 point: the reason it is the same for every order, so every code passes the test for 9. (b) 1 point: no code is divisible by 3, because every order has digit sum 17.

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6. Hiro runs the sieve on a board from 1 to 100. (3 points)

(a) The lanterns round 3 puts out are 9, 15, 21, 27, … Explain why they go up in sixes and not in threes.

(b) How many lanterns does round 3 put out on his board?

(c) Which of the lanterns 57, 75 and 85 does round 3 put out?

ANSWER

(a) The multiples of 3 take turns being odd and even, and the even ones went out in round 2, so only every other multiple of 3 is still lit, and those are 6 apart. (b) 16 (c) 57 and 75

SOLUTION

(a) The multiples of 3 after 3 are 6, 9, 12, 15, …: even, odd, even, odd. Round 2 already put out the even ones. So round 3 finds only every other multiple of 3 still lit, the odd ones, and two steps of 3 make 6.

(b) The odd multiples of 3 from 9 to 99: 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99. That is 16 lanterns. (15, 45 and 75 are on the list: round 3 comes before round 5, so it reaches them first.)

(c) 57 = 3 × 19 and 75 = 3 × 25 are odd multiples of 3, so round 3 puts them out; round 5 later finds 75 already out. 85 = 3 × 28 + 1 is not a multiple of 3; 85 = 5 × 17 goes out in round 5.

SCORING

1 point for explaining that the even multiples of 3 are already out, so only every other one is new; 1 point for 16; 1 point for 57 and 75 (not 85).

7. Part of a ladder has been rubbed out. The letters A, B, C and D stand for the missing entries. (2 points)

Divide by	Number
2	A
B	805
7	C
D	23
	1

(a) Find A, B, C and D.

(b) Write the prime factorization of A, smallest prime first.

ANSWER

(a) $A = 1{,}610$, $B = 5$, $C = 161$, $D = 23$ (b) $1{,}610 = 2 \cdot 5 \cdot 7 \cdot 23$

SOLUTION

(a) Each line of a ladder says that its number equals the prime beside it times the number below. Work from the bottom up.

The last line divides down to 1, so $D = 23$. Next, $C = 7 \cdot 23 = 161$. Then $805 = B \cdot 161$, so $B = 805 \div 161 = 5$, which is prime, as every divisor on a ladder must be. Finally $A = 2 \cdot 805 = 1{,}610$.

(b) The primes down the left side are 2, 5, 7 and 23, each used once, so $1{,}610 = 2 \cdot 5 \cdot 7 \cdot 23$. Check: $2 \cdot 5 \cdot 161 = 1{,}610$.

SCORING

1 point for all four of A, B, C and D; 1 point for the prime factorization of A.

8. Mia is running the sieve on a board from 1 to 70. The board shows her work so far: she put out 1, circled 2 and 5, and shaded and struck through every number she has put out. She has made a mistake. (3 points)

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~~1~~	(2)	3	~~4~~	(5)	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
21	~~22~~	23	~~24~~	~~25~~	~~26~~	27	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	~~35~~	~~36~~	37	~~38~~	39	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
51	~~52~~	53	~~54~~	~~55~~	~~56~~	57	~~58~~	59	~~60~~
61	~~62~~	63	~~64~~	~~65~~	~~66~~	67	~~68~~	69	~~70~~

(a) Which round did Mia skip? Name one lit number that shows it.

(b) Mia runs the skipped round. Which numbers does it put out?

(c) After that, which rounds does the board still need?

ANSWER

(a) Round 3; for example $9 = 3 \times 3$ is still lit. (b) 9, 21, 27, 33, 39, 51, 57, 63, 69 (c) Only round 7, which puts out 49.

SOLUTION

(a) The struck-through numbers are 1, the even numbers after 2 and the multiples of 5 after 5, so Mia ran rounds 2 and 5. The board needs round 3 before round 5, and it is missing: $9 = 3 \times 3$ is still lit (so are 21, 27 and others).

(b) Round 3 puts out the multiples of 3 after 3 that are still lit: the odd multiples of 3, going up in sixes from 9, except those ending in 5. That is 9, 21, 27, 33, 39, 51, 57, 63, 69 (15 and 45 are already out from round 5).

(c) $7 \times 7 = 49 \leq 70$ but $11 \times 11 = 121 > 70$, so the board needs round 7 and then stops. Round 7 puts out only 49: $7 \times 11 = 77$ is off the board, and 14 to 42 and 56, 63, 70 are already out.

SCORING

1 point for round 3 with a lit multiple of 3 as evidence; 1 point for the nine numbers 9, 21, 27, 33, 39, 51, 57, 63, 69 (no point if 15 or 45 is listed or one is missing); 1 point for round 7 as the only round left, putting out 49.

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