# Math 6 Unit 1 quiz 2: Every number has exactly one prime factorization, Greatest common factor by listing, Least common multiple by listing (answer key)

8 questions, 22 points

**ANSWERS AT A GLANCE**

1. (a) 9 (b) 8 (c) 17
2. (a) 56 (b) 45 (c) 44
3. 10, 20, 40, 50, 100, 200
4. No. An even factor would put a 2 into the prime factorization of 81, and $81 = 3^4$ has no 2.
5. $2^3 \cdot 3^2 \cdot 5^2$. It is not a perfect square.
6. 4 sets, each with 2 motors, 4 wheels and 9 wires.
7. 4 m long, 32 pieces.
8. After 33 beats; Zoe has hit the drum 11 times.

**WORKED SOLUTIONS**

**1.** Find the greatest common factor by listing factors. (3 points)

(a) 27 and 63 (b) 40 and 88 (c) 34 and 51

**ANSWER**

(a) 9 (b) 8 (c) 17

**SOLUTION**

(a) Factors of 27: 1, 3, 9, 27. 27 does not divide 63; 63 = 9 × 7. GCF(27, 63) = 9.

(b) Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40. 88 ÷ 40, 88 ÷ 20 and 88 ÷ 10 leave remainders; 88 = 8 × 11. GCF(40, 88) = 8.

(c) Factors of 34: 1, 2, 17, 34. 51 is odd, so 34 fails; 51 = 17 × 3. GCF(34, 51) = 17.

**SCORING**

1 point for each correct GCF, with a factor list or a test from the top that supports it.

**2.** Find the least common multiple. (3 points)

(a) 8 and 14 (b) 9 and 15 (c) 11 and 44

**ANSWER**

(a) 56 (b) 45 (c) 44

**SOLUTION**

(a) Multiples of 14: 14, 28 and 42 leave remainders when divided by 8, and 56 = 8 × 7. So LCM(8, 14) = 56.

(b) Multiples of 15: 15 and 30 leave remainders when divided by 9, and 45 = 9 × 5. So LCM(9, 15) = 45.

(c) 44 ÷ 11 = 4 exactly, so LCM(11, 44) = 44.

**SCORING**

1 point for each correct least common multiple, with a list or a division test shown.

**3.** $m = 2^3 \cdot 5^2$. Using the prime factorization, list every factor of $m$ that is a multiple of 10. (2 points)

**ANSWER**

10, 20, 40, 50, 100, 200

**SOLUTION**


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A multiple of $10 = 2 \cdot 5$ needs at least one 2 and at least one 5. A factor of $m$ has at most three 2s and at most two 5s. So the factors that are multiples of 10 are $2^i \cdot 5^j$ with $i$ one of 1, 2, 3 and $j$ one of 1, 2:

$2 \cdot 5 = 10$, $4 \cdot 5 = 20$, $8 \cdot 5 = 40$, $2 \cdot 25 = 50$, $4 \cdot 25 = 100$, $8 \cdot 25 = 200$.

**4.** Mateo says he found two whole numbers, both greater than 1, that multiply to $3^4 = 81$, and one of them is even. Is that possible? Explain using prime factorizations. (2 points)

**ANSWER**

No. An even factor would put a 2 into the prime factorization of 81, and $81 = 3^4$ has no 2.

**SOLUTION**

Suppose $81 = a \cdot b$ with $a$ even. Then $a = 2 \cdot k$ for some whole number $k$. Factorizing $k$ and $b$ and writing a 2 in front gives a prime factorization of 81 that contains a 2. But $81 = 3^4$ contains no 2, and by uniqueness 81 has only one prime factorization. So it is not possible. (The factor pairs are $3 \cdot 27$ and $9 \cdot 9$, all odd.)

**5.** Without multiplying the numbers out, write $6 \cdot 15 \cdot 20$ as a prime factorization in tidy form. Then decide whether the product is a perfect square, and if it is, of which whole number. (3 points)

**ANSWER**

$2^3 \cdot 3^2 \cdot 5^2$. It is not a perfect square.

**SOLUTION**

Split each number into primes: $6 = 2 \cdot 3$, $15 = 3 \cdot 5$, $20 = 2^2 \cdot 5$. Gathering them gives $2^3 \cdot 3^2 \cdot 5^2$.

The power of 2 is odd. A square of a whole number has every power even, and by uniqueness this is the only factorization, so the product is not a perfect square.

**SCORING**

1 point: each of the three numbers split into primes correctly. 1 point: the tidy form gathered correctly. 1 point: the square decision read from the powers (even or not), with the whole number named when it is a square.

**6.** A robotics club has 8 motors, 16 wheels and 36 wires. It packs all of them into identical building sets with nothing left over. What is the largest number of sets it can pack, and what goes in each set? (3 points)

**ANSWER**

4 sets, each with 2 motors, 4 wheels and 9 wires.

**SOLUTION**

The number of sets must divide all three amounts, so the most sets is GCF(8, 16, 36). Work down the factors of 8, the smallest amount: the first that divides both 16 and 36 is 4, since $8 = 4 \times 2$, $16 = 4 \times 4$ and $36 = 4 \times 9$, and 2, 4 and 9 share no factor but 1.

The club packs 4 sets, each with 2 motors, 4 wheels and 9 wires.

**SCORING**

1 point for explaining that the number of sets must divide every amount (so it is the GCF). 1 point for the correct number of sets. 1 point for the correct contents of each set.

**7.** Leah has two coils of craft wire, one 48 m long and one 80 m long. She cuts all of it into pieces of the same length, a whole number of meters, with none wasted. Her class needs at least 20 pieces. What is the longest she can make the pieces, and how many pieces will she get? (3 points)


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**ANSWER**
4 m long, 32 pieces.

**SOLUTION**
Each coil is cut into equal pieces with none over, so the piece length is a common factor of 48 and 80. Work down the factors of 48: 80 ÷ 48 and 80 ÷ 24 leave remainders, but 80 = 16 × 5, so GCF(48, 80) = 16. Every common factor is a factor of 16: 1, 2, 4, 8, 16.

Try them from the top. Pieces of 16 m give 3 + 5 = 8 pieces, too few. Pieces of 8 m give 6 + 10 = 16, still too few. Pieces of 4 m give 12 + 20 = 32 pieces, enough. So the pieces are 4 m long and she gets 32 pieces.

**SCORING**
1 point for seeing that the piece length must be a common factor of 48 and 80 (the factors of 16). 1 point for testing from the top and rejecting 16 and 8 because they give fewer than 20 pieces. 1 point for 4 m and 32 pieces.

8. In a drumming class, Zoe hits a drum every 3 beats and the teacher rings a bell every 11 beats. They sound together on the first count. How many beats later do they next sound together, and how many times has Zoe hit the drum by then, not counting the first count? (3 points)

**ANSWER**
After 33 beats; Zoe has hit the drum 11 times.

**SOLUTION**
The drum sounds at the multiples of 3 beats and the bell at the multiples of 11, so they next sound together after LCM(3, 11) beats. List the multiples of 11 and test each by dividing by 3: the first that divides exactly is 33 = 3 × 11.

By then Zoe has hit the drum 33 ÷ 3 = 11 times.

**SCORING**
1 point for seeing that the answer is the least common multiple of the two counts; 1 point for the correct number of beats; 1 point for the number of drum hits.


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