Math 6 Unit 1 worksheet: Tests for three and nine (answer key)

ANSWERS AT A GLANCE

1. (a) digit sum 21, remainder 3 by 9, 0 by 3 (b) digit sum 18, remainder 0 by 9, 0 by 3 (c) digit sum 13, remainder 4 by 9, 1 by 3 (d) digit sum 26, remainder 8 by 9, 2 by 3 (e) digit sum 6, remainder 6 by 9, 0 by 3 (f) digit sum 23, remainder 5 by 9, 2 by 3
2. On the last page: 5. More to fill it: 4.
3. (a) Divisible by 3, not by 9. (b) 3 (c) 3,999
4. 98,730
5. The correct answer is 825 remainder 1.
6. The digit 3, written anywhere: for example 32,571 or 25,713.

WORKED SOLUTIONS

1. For each number, find its digit sum. Then use the digit sum to find the remainder when the number is divided by 9 and when it is divided by 3.

(a) 4,827 (b) 63,018 (c) 7,051 (d) 95,417 (e) 300,003 (f) 81,725

ANSWER

(a) digit sum 21, remainder 3 by 9, 0 by 3 (b) digit sum 18, remainder 0 by 9, 0 by 3 (c) digit sum 13, remainder 4 by 9, 1 by 3 (d) digit sum 26, remainder 8 by 9, 2 by 3 (e) digit sum 6, remainder 6 by 9, 0 by 3 (f) digit sum 23, remainder 5 by 9, 2 by 3

SOLUTION

A number leaves the same remainder as its digit sum, both on division by 9 and on division by 3.

(a) $4 + 8 + 2 + 7 = 21$. Since $21 = 9 \times 2 + 3$ and $21 = 3 \times 7$, the remainders are 3 by 9 and 0 by 3.

(b) $6 + 3 + 0 + 1 + 8 = 18 = 9 \times 2$, so the remainder is 0 by 9, and 0 by 3 as well.

(c) $7 + 0 + 5 + 1 = 13$. Since $13 = 9 + 4$ and $13 = 3 \times 4 + 1$, the remainders are 4 by 9 and 1 by 3.

(d) $9 + 5 + 4 + 1 + 7 = 26$. Since $26 = 9 \times 2 + 8$ and $26 = 3 \times 8 + 2$, the remainders are 8 by 9 and 2 by 3.

(e) $3 + 0 + 0 + 0 + 0 + 3 = 6$. Since 6 is less than 9, the remainder by 9 is 6; since $6 = 3 \times 2$, the remainder by 3 is 0.

(f) $8 + 1 + 7 + 2 + 5 = 23$. Since $23 = 9 \times 2 + 5$ and $23 = 3 \times 7 + 2$, the remainders are 5 by 9 and 2 by 3.

2. Mateo puts 2,417 stamps into an album. Each page holds 9 stamps, and a page is filled before the next one is started, so every page is full except the last one. Use the digit sum to find how many stamps are on the last page, and how many more stamps would fill it.

ANSWER

On the last page: 5. More to fill it: 4.

SOLUTION

The digit sum of 2,417 is $2 + 4 + 1 + 7 = 14$. Since $14 = 9 \times 1 + 5$, the digit sum leaves remainder 5 when divided by 9, and a number leaves the same remainder as its digit sum. So the stamps fill whole pages of 9, and the number on the last page is the remainder, 5. A full page needs $9 - 5 = 4$ more.

Check: $9 \times 268 + 5 = 2{,}417$.

3. Jenny thinks of a whole number whose digits add up to 30. She does not say what the number is.

(a) Is her number divisible by 3? Is it divisible by 9?

(b) What remainder does her number leave when it is divided by 9?

(c) What is the smallest four-digit number whose digit sum is 30?

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ANSWER
(a) Divisible by 3, not by 9. (b) 3 (c) 3,999

SOLUTION
(a) The test only needs the digit sum, so it does not matter which number Jenny chose. $30 = 3 \times 10$ is divisible by 3, so her number is divisible by 3. 30 is not divisible by 9, so her number is not.

(b) $30 = 9 \times 3 + 3$, so the digit sum leaves remainder 3, and so does her number.

(c) Three digits add to at most $9 + 9 + 9 = 27$, so the first digit must be at least $30 - 27 = 3$. With first digit 3, the other three digits must add to 27, so each of them is 9. The smallest such number is 3,999. **Check:** $3 + 9 + 9 + 9 = 30$, and $3{,}999 = 3 \times 1{,}333$, as part (a) says it must be.

4. Find the largest five-digit number that is divisible by 9 and has five different digits. Explain how you know that no larger number works.

ANSWER
98,730

SOLUTION
The largest five-digit number with different digits is 98,765, but its digit sum is $35 = 9 \times 3 + 8$, so it is not divisible by 9.

Keep the first three digits as large as possible: 9, 8 and 7, which add to 24. The last two digits must be different from these and from each other, so each is 6 or less, and together they add to at most $6 + 5 = 11$. The whole digit sum is then between 24 and 35, and the only multiple of 9 there is 27, so the last two digits must add to 3. The largest choice puts 3 in the fourth place and 0 in the last: 98,730.

Any larger five-digit number with different digits also starts 987 (since 98,765 is the largest), and its last two digits make a two-digit number larger than 30, so they add to at least 4 and its digit sum is not 27. **Check:** $9 + 8 + 7 + 3 + 0 = 27$, and $98{,}730 = 9 \times 10{,}970$.

5. Omar divides 7,426 by 9 and writes his answer as 824 remainder 6. Without dividing again, use the digit sum to show that his remainder is wrong. Then find the correct remainder and the correct quotient.

ANSWER
The correct answer is 825 remainder 1.

SOLUTION
The digit sum of 7,426 is $7 + 4 + 2 + 6 = 19$, and $19 = 9 \times 2 + 1$. A number leaves the same remainder as its digit sum, so 7,426 leaves remainder 1 when divided by 9, not 6.

Omar's quotient was too small by one: $9 \times 824 = 7{,}416$, and $7{,}426 - 7{,}416 = 10$, which is one more group of 9 with 1 over. So the correct answer is 825 remainder 1. **Check:** $9 \times 825 + 1 = 7{,}425 + 1 = 7{,}426$.

6. Mia has the number 2,571. She wants to write one more digit somewhere in it (in front, between two of its digits, or at the end) so that the new five-digit number is divisible by 9. Which digit must she write? Give two different numbers she could make.

ANSWER
The digit 3, written anywhere: for example 32,571 or 25,713.

SOLUTION

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The digit sum of 2,571 is 2 + 5 + 7 + 1 = 15. Writing a new digit adds that digit to the digit sum, wherever it goes, so the new digit sum is 15 plus a digit: somewhere from 15 to 24. The only multiple of 9 in that range is 18, so the new digit must be 18 − 15 = 3.

The place does not matter, since adding the digits in a different order gives the same sum. So 32,571, 23,571, 25,371, 25,731 and 25,713 all work. **Check:** 32,571 = 9 × 3,619 and 25,713 = 9 × 2,857.

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