Math 6 Unit 1 worksheet: Prime factorization (answer key)

ANSWERS AT A GLANCE

1. (a) $72 = 2^3 \cdot 3^2$ (b) $150 = 2 \cdot 3 \cdot 5^2$ (c) $156 = 2^2 \cdot 3 \cdot 13$ (d) $231 = 3 \cdot 7 \cdot 11$ (e) $288 = 2^5 \cdot 3^2$ (f) $500 = 2^2 \cdot 5^3$
2. (a) $252 = 2^2 \cdot 3^2 \cdot 7$ (b) $325 = 5^2 \cdot 13$ (c) $1{,}078 = 2 \cdot 7^2 \cdot 11$ (d) $2{,}025 = 3^4 \cdot 5^2$ (e) $1{,}716 = 2^2 \cdot 3 \cdot 11 \cdot 13$ (f) $391 = 17 \cdot 23$
3. The missing exponent is 2: $396 = 2^2 \cdot 3^2 \cdot 11$.
4. (a) $105 = 3 \cdot 5 \cdot 7$ (b) 3 by 5 by 7 bars (c) The three edges multiply to 105, so each edge is a product of some of its primes; each edge longer than 1 needs at least one prime, and there are only three primes, so each edge gets exactly one.
5. (a) $32 = 2^5$ (b) $900 = 2^2 \cdot 3^2 \cdot 5^2$
6. (a) $C = 9$, $B = 18$, $A = 630$ (b) $630 = 2 \cdot 3^2 \cdot 5 \cdot 7$
7. (a) 4 and 9 are not prime, so her answer is not a prime factorization; the ladder divides by primes only. (b) $1{,}260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$

WORKED SOLUTIONS

1. Use a factor tree to find the prime factorization. Write it with exponents, smallest prime first, and check by multiplying.

(a) 72 (b) 150 (c) 156

(d) 231 (e) 288 (f) 500

ANSWER

(a) $72 = 2^3 \cdot 3^2$ (b) $150 = 2 \cdot 3 \cdot 5^2$ (c) $156 = 2^2 \cdot 3 \cdot 13$ (d) $231 = 3 \cdot 7 \cdot 11$ (e) $288 = 2^5 \cdot 3^2$ (f) $500 = 2^2 \cdot 5^3$

SOLUTION

Any first split works; one tree is shown for each.

(a) $72 = 8 \cdot 9$, $8 = 2 \cdot 4$, $4 = 2 \cdot 2$ and $9 = 3 \cdot 3$. The circled primes are three 2s and two 3s: $72 = 2^3 \cdot 3^2$. Check: $8 \cdot 9 = 72$.

(b) $150 = 10 \cdot 15$, $10 = 2 \cdot 5$ and $15 = 3 \cdot 5$: $150 = 2 \cdot 3 \cdot 5^2$. Check: $6 \cdot 25 = 150$.

(c) $156 = 12 \cdot 13$, and 13 is prime. $12 = 3 \cdot 4$ and $4 = 2 \cdot 2$: $156 = 2^2 \cdot 3 \cdot 13$. Check: $4 \cdot 39 = 156$.

(d) The digits of 231 add to 6, so $231 = 3 \cdot 77$, and $77 = 7 \cdot 11$: $231 = 3 \cdot 7 \cdot 11$. Each prime appears once, so there are no exponents. Check: $21 \cdot 11 = 231$.

(e) $288 = 16 \cdot 18$, $16 = 4 \cdot 4$ with each $4 = 2 \cdot 2$, and $18 = 2 \cdot 9$ with $9 = 3 \cdot 3$. That is four 2s and one more, five in all, and two 3s: $288 = 2^5 \cdot 3^2$. Check: $32 \cdot 9 = 288$.

(f) $500 = 5 \cdot 100$, $100 = 10 \cdot 10$ and each $10 = 2 \cdot 5$: two 2s and three 5s, so $500 = 2^2 \cdot 5^3$. Check: $4 \cdot 125 = 500$.

2. Use the ladder to find the prime factorization. Write it with exponents, smallest prime first.

(a) 252 (b) 325 (c) 1,078

(d) 2,025 (e) 1,716 (f) 391

ANSWER

(a) $252 = 2^2 \cdot 3^2 \cdot 7$ (b) $325 = 5^2 \cdot 13$ (c) $1{,}078 = 2 \cdot 7^2 \cdot 11$ (d) $2{,}025 = 3^4 \cdot 5^2$ (e) $1{,}716 = 2^2 \cdot 3 \cdot 11 \cdot 13$ (f) $391 = 17 \cdot 23$

SOLUTION

(a) $252 \div 2 = 126$, $126 \div 2 = 63$, $63 \div 3 = 21$, $21 \div 3 = 7$, $7 \div 7 = 1$. So $252 = 2^2 \cdot 3^2 \cdot 7$. Check: $4 \cdot 63 = 252$.

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(b) 325 is odd and its digits add to 10, so 2 and 3 fail. It ends in 5: 325 ÷ 5 = 65, 65 ÷ 5 = 13, 13 ÷ 13 = 1. So $325 = 5^2 \cdot 13$. Check: 25 · 13 = 325.

(c) 1,078 ÷ 2 = 539. The digits of 539 add to 17 and it ends in 9, so 3 and 5 fail. 539 ÷ 7 = 77, 77 ÷ 7 = 11, 11 ÷ 11 = 1. So $1{,}078 = 2 \cdot 7^2 \cdot 11$. Check: 2 · 49 · 11 = 1,078.

(d) 2,025 is odd and its digits add to 9. 2,025 ÷ 3 = 675, 675 ÷ 3 = 225, 225 ÷ 3 = 75, 75 ÷ 3 = 25; the digits of 25 add to 7, so move on to 5: 25 ÷ 5 = 5, 5 ÷ 5 = 1. So $2{,}025 = 3^4 \cdot 5^2$. Check: 81 · 25 = 2,025.

(e) 1,716 ÷ 2 = 858, 858 ÷ 2 = 429. The digits of 429 add to 15: 429 ÷ 3 = 143. Then 3, 5 and 7 fail on 143 (7 · 20 = 140 leaves 3), and 143 ÷ 11 = 13, 13 ÷ 13 = 1. So $1{,}716 = 2^2 \cdot 3 \cdot 11 \cdot 13$. Check: 12 · 143 = 1,716.

(f) 391 is odd, its digits add to 13, and it ends in 1, so 2, 3 and 5 fail. 7 · 55 = 385 leaves 6; 11 · 35 = 385 leaves 6; 13 · 30 = 390 leaves 1. Then 391 ÷ 17 = 23. Only 2 and 3 need trying on 23 (since 5 · 5 = 25), and both fail, so 23 is prime: 23 ÷ 23 = 1. So 391 = 17 · 23. Check: 17 · 23 = 391.

3. Fill in the missing exponent: $396 = 2^{\square} \cdot 3^2 \cdot 11$.

ANSWER

The missing exponent is 2: $396 = 2^2 \cdot 3^2 \cdot 11$.

SOLUTION

The known part is $3^2 \cdot 11 = 9 \cdot 11 = 99$. Divide it out: 396 ÷ 99 = 4.

So $2^{\square} = 4$. Halving 4 again and again reaches 1 after 2 halvings, so 4 is 2 twos multiplied together: $4 = 2^2$.

Check: 4 · 99 = 396.

4. Mateo packs 105 bars of soap into a box as one solid block, so many bars long, so many wide and so many high. No edge of the block is just 1 bar long.

(a) Find the prime factorization of 105.

(b) How many bars long, wide and high is the block?

(c) Explain why there is no other way to do it, apart from turning the box around.

ANSWER

(a) 105 = 3 · 5 · 7 (b) 3 by 5 by 7 bars (c) The three edges multiply to 105, so each edge is a product of some of its primes; each edge longer than 1 needs at least one prime, and there are only three primes, so each edge gets exactly one.

SOLUTION

(a) 105 ends in 5: 105 = 5 · 21, and 21 = 3 · 7. So 105 = 3 · 5 · 7.

(b) The block holds length times width times height bars, so the three edges multiply to 105: a block 3 by 5 by 7 works, since 3 · 5 · 7 = 105.

(c) Split each edge into its primes. Together these primes make a prime factorization of 105, and 105 always comes out as 3 · 5 · 7. An edge longer than 1 bar is not 1, so it uses at least one of these primes. Three edges and only three primes means each edge gets exactly one prime, so the edges are 3, 5 and 7 in some order.

5. (a) Find the smallest whole number whose factor tree ends in exactly five circled primes (count a prime each time it is circled).

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(b) Find the smallest whole number whose prime factorization uses three different primes, each with exponent 2.

Explain each answer.

ANSWER

(a) $32 = 2^5$ (b) $900 = 2^2 \cdot 3^2 \cdot 5^2$

SOLUTION

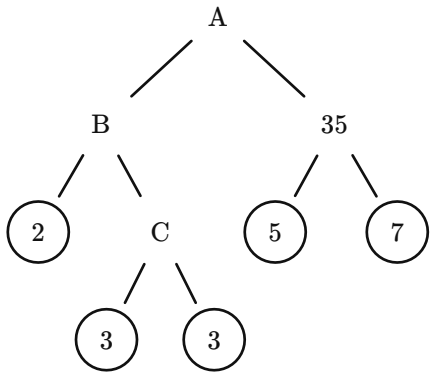
(a) The number is the product of its five circled primes. The smallest prime is 2, and using a bigger prime anywhere makes the product bigger, so the smallest choice is five 2s: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.

(b) The number is $p^2 \cdot q^2 \cdot r^2$ for three different primes, and it is smallest when the primes are the three smallest: 2, 3 and 5. So it is $2^2 \cdot 3^2 \cdot 5^2 = 4 \cdot 9 \cdot 25 = 900$.

6. Jasmine's factor tree has lost three of its numbers, marked A, B and C. The circled numbers are prime.

(a) Find C, B and A.

(b) Write the prime factorization of A with exponents, smallest prime first.



ANSWER

(a) $C = 9$, $B = 18$, $A = 630$ (b) $630 = 2 \cdot 3^2 \cdot 5 \cdot 7$

SOLUTION

(a) Each number on the tree is the product of the two numbers below it. Work from the bottom up: $C = 3 \cdot 3 = 9$, then $B = 2 \cdot C = 2 \cdot 9 = 18$, then $A = B \cdot 35 = 18 \cdot 35 = 630$.

(b) The circled numbers are 2, 3, 3, 5 and 7, so $630 = 2 \cdot 3^2 \cdot 5 \cdot 7$. Check: 2 · 9 · 35 = 630.

7. Emma used a ladder on 1,260 and wrote this:

Divide by	Number
4	1,260
9	315
5	35
7	7
	1

She wrote 1,260 = 4 · 9 · 5 · 7. Every division is correct.

(a) Explain what is wrong with her answer.

(b) Find the prime factorization of 1,260, written with exponents.

ANSWER

(a) 4 and 9 are not prime, so her answer is not a prime factorization; the ladder divides by primes only. (b) $1{,}260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$

SOLUTION

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(a) Her product is right, $4 \cdot 9 \cdot 5 \cdot 7 = 1{,}260$, but a prime factorization may contain only primes, and $4 = 2 \cdot 2$ and $9 = 3 \cdot 3$ are not prime. Trying the primes in order from 2 would have stopped this: by the time 4 is reached, every 2 is already divided out.

(b) Split the two composite numbers: $1{,}260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$. A correct ladder gives the same: $1{,}260 \div 2 = 630$, $630 \div 2 = 315$, $315 \div 3 = 105$, $105 \div 3 = 35$, $35 \div 5 = 7$, $7 \div 7 = 1$. Check: $4 \cdot 9 \cdot 35 = 1{,}260$.

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