Math 6 Unit 1 worksheet: Finding them from prime factorizations (answer key)

ANSWERS AT A GLANCE

1. (a) GCF $2^2 \cdot 5 = 20$, LCM $2^3 \cdot 5^2 \cdot 7 = 1{,}400$ (b) GCF $3^2 \cdot 7 = 63$, LCM $3^4 \cdot 7^3 = 27{,}783$ (c) GCF $2 \cdot 3 = 6$, LCM $2^3 \cdot 3^2 \cdot 11 \cdot 13 = 10{,}296$ (d) GCF 1, LCM $2^4 \cdot 3 \cdot 5^3 = 6{,}000$
2. GCF = 18, LCM = 1,980
3. GCF = 24, LCM = 720
4. (a) $150 \cdot 32 = 4{,}800$ and $150 \cdot 33 = 4{,}950$, so 150 does not go into 4,900. (b) She left out the 3, which is in 150 only. The LCM is $2^2 \cdot 3 \cdot 5^2 \cdot 7^2 = 14{,}700$.
5. (a) 90 (b) 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
6. (a) It passes: $84 \cdot 90 = 7{,}560$ and $6 \cdot 1{,}260 = 7{,}560$. (b) It fails: $54 \cdot 120 = 6{,}480$ but $6 \cdot 540 = 3{,}240$. The LCM is 1,080. (c) No. The test only checks the product of the two answers; a wrong pair such as GCF 3 and LCM 2,520 also multiplies to 7,560.
7. 360 jars: 15 cases of 24, or 8 cases of 45.

WORKED SOLUTIONS

1. Find the GCF and the LCM of each pair. Give each answer as a product of prime powers and as an ordinary number.

(a) $2^3 \cdot 5^2$ and $2^2 \cdot 5 \cdot 7$

(b) $3^4 \cdot 7$ and $3^2 \cdot 7^3$

(c) $2 \cdot 3^2 \cdot 13$ and $2^3 \cdot 3 \cdot 11$

(d) 5^3 and $2^4 \cdot 3$

ANSWER

(a) GCF $2^2 \cdot 5 = 20$, LCM $2^3 \cdot 5^2 \cdot 7 = 1{,}400$ (b) GCF $3^2 \cdot 7 = 63$, LCM $3^4 \cdot 7^3 = 27{,}783$ (c) GCF $2 \cdot 3 = 6$, LCM $2^3 \cdot 3^2 \cdot 11 \cdot 13 = 10{,}296$ (d) GCF 1, LCM $2^4 \cdot 3 \cdot 5^3 = 6{,}000$

SOLUTION

(a) Shared primes: 2 (powers 2^3 and 2^2) and 5 (powers 5^2 and 5^1). The 7 is in the second number only. GCF $= 2^2 \cdot 5 = 20$. LCM $= 2^3 \cdot 5^2 \cdot 7 = 8 \cdot 25 \cdot 7 = 1{,}400$.

(b) Both numbers use only 3 and 7. Smaller powers: 3^2 and 7^1, so GCF = $9 \cdot 7 = 63$. Greater powers: 3^4 and 7^3, so LCM = $81 \cdot 343 = 27{,}783$.

(c) Shared primes: 2 (smaller power 2^1) and 3 (smaller power 3^1); 13 and 11 are each in one number only. GCF = $2 \cdot 3 = 6$. LCM $= 2^3 \cdot 3^2 \cdot 11 \cdot 13 = 72 \cdot 143 = 10{,}296$.

(d) No prime is in both numbers, so GCF = 1. The LCM takes every prime at its own power: $2^4 \cdot 3 \cdot 5^3 = 16 \cdot 3 \cdot 125 = 6{,}000$, which is just the product of the two numbers.

2. Write 198 and 180 as products of primes. Then use the factorizations to find their GCF and their LCM.

ANSWER

GCF = 18, LCM = 1,980

SOLUTION

Factorize each number:

$$198 = 2^1 \cdot 3^2 \cdot 11 \qquad 180 = 2^2 \cdot 3^2 \cdot 5$$

The primes 2 and 3 are in both numbers, and 11 and 5 are each in one number only, so they stay out of the GCF. The GCF takes the smaller power of each shared prime:

$$\text{GCF} = 2^1 \cdot 3^2 = 18$$

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The LCM takes the greater power of every prime in either number:

$$\text{LCM} = 2^2 \cdot 3^2 \cdot 11 \cdot 5 = 1{,}980$$

3. Find the GCF and the LCM of 48, 72 and 120 from their prime factorizations.

ANSWER

GCF = 24, LCM = 720

SOLUTION

$$48 = 2^4 \cdot 3 \qquad 72 = 2^3 \cdot 3^2 \qquad 120 = 2^3 \cdot 3 \cdot 5$$

For the GCF a prime must be in all three. The 2 and the 3 are; the 5 is only in 120. The smallest powers are 2^3 and 3^1, so GCF $= 2^3 \cdot 3 = 24$.

For the LCM take the greatest power of every prime: 2^4, 3^2 and 5^1, so LCM = $16 \cdot 9 \cdot 5 = 720$.

Check: $720 \div 48 = 15$, $720 \div 72 = 10$, $720 \div 120 = 6$, and $48 \div 24 = 2$, $72 \div 24 = 3$, $120 \div 24 = 5$.

4. Sarah finds the LCM of 150 and 196. She writes $150 = 2 \cdot 3 \cdot 5^2$ and $196 = 2^2 \cdot 7^2$, and then LCM $= 2^2 \cdot 5^2 \cdot 7^2 = 4{,}900$.

(a) Without factorizing anything, show that 4,900 cannot be a common multiple of 150 and 196.

(b) What did Sarah do wrong? Find the correct LCM.

ANSWER

(a) $150 \cdot 32 = 4{,}800$ and $150 \cdot 33 = 4{,}950$, so 150 does not go into 4,900. (b) She left out the 3, which is in 150 only. The LCM is $2^2 \cdot 3 \cdot 5^2 \cdot 7^2 = 14{,}700$.

SOLUTION

(a) A common multiple of 150 and 196 must be a multiple of 150. But $150 \cdot 32 = 4{,}800$ and $150 \cdot 33 = 4{,}950$, and 4,900 lies between them, so 4,900 is not a multiple of 150.

(b) Her factorizations are right. The LCM takes the greater power of every prime in either number, including a prime found in only one number. The 3 is in 150 but not in 196, and she dropped it. Correctly:

$$\text{LCM} = 2^2 \cdot 3 \cdot 5^2 \cdot 7^2 = 4 \cdot 3 \cdot 25 \cdot 49 = 14{,}700$$

Check: $14{,}700 \div 150 = 98$ and $14{,}700 \div 196 = 75$.

5. (a) Find GCF(270, 630) from the prime factorizations.

(b) Every common factor of two numbers is a factor of their GCF. Use this to list all the common factors of 270 and 630.

ANSWER

(a) 90 (b) 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

SOLUTION

(a) $270 = 2 \cdot 3^3 \cdot 5$ and $630 = 2 \cdot 3^2 \cdot 5 \cdot 7$. The shared primes are 2, 3 and 5, at the smaller powers 2^1, 3^2 and 5^1; the 7 is only in 630. So GCF = $2 \cdot 9 \cdot 5 = 90$.

(b) The common factors are exactly the factors of 90. In pairs: $1 \cdot 90$, $2 \cdot 45$, $3 \cdot 30$, $5 \cdot 18$, $6 \cdot 15$, $9 \cdot 10$. So the common factors are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90, twelve in all. Listing every factor of 270

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(there are 16) and of 630 (there are 24) and comparing would take far longer.

6. Use the product rule, not new factorizations, to test each claim.

(a) Mia says GCF(84, 90) = 6 and LCM(84, 90) = 1,260.

(b) Ryan says GCF(54, 120) = 6 and LCM(54, 120) = 540. His GCF is right. If his claim fails the test, find the correct LCM.

(c) Does passing the test prove that both of Mia's numbers are right? Give a reason.

ANSWER

(a) It passes: 84 · 90 = 7,560 and 6 · 1,260 = 7,560. (b) It fails: 54 · 120 = 6,480 but 6 · 540 = 3,240. The LCM is 1,080. (c) No. The test only checks the product of the two answers; a wrong pair such as GCF 3 and LCM 2,520 also multiplies to 7,560.

SOLUTION

(a) The product rule says GCF · LCM must equal the product of the two numbers. 84 · 90 = 7,560 and 6 · 1,260 = 7,560, so the claim passes.

(b) 54 · 120 = 6,480, but 6 · 540 = 3,240, only half as much, so the claim fails. Since the GCF is right, the LCM is the product divided by the GCF. Divide first: $\frac{54}{6} \cdot 120 = 9 \cdot 120 = 1{,}080$. (Check: $54 = 2 \cdot 3^3$ and $120 = 2^3 \cdot 3 \cdot 5$ give LCM $= 2^3 \cdot 3^3 \cdot 5 = 1{,}080$.)

(c) No. The test only says the two answers multiply to the right product. A GCF of 3 with an LCM of 2,520 multiplies to 7,560 too, and both of those are wrong. To be sure, check at least one of the two another way: here $84 = 2^2 \cdot 3 \cdot 7$ and $90 = 2 \cdot 3^2 \cdot 5$ give GCF = 2 · 3 = 6, and then the test forces the LCM to be 1,260.

7. Priya keeps bees and puts the honey from one harvest into jars. All the jars can be packed into full cases of 24 jars, or instead into full cases of 45 jars, with no jar left over either way. What is the smallest number of jars there could be, and how many cases does each way use?

ANSWER

360 jars: 15 cases of 24, or 8 cases of 45.

SOLUTION

The number of jars is a multiple of 24 and also a multiple of 45, so the smallest it can be is the LCM. Factorize:

$$24 = 2^3 \cdot 3 \qquad 45 = 3^2 \cdot 5$$

Take the greater power of every prime that appears in either number: LCM = 360. So there are 360 jars, which fill 360 ÷ 24 = 15 cases of 24 or 360 ÷ 45 = 8 cases of 45. Size check: 360 is no smaller than 45, and the smaller cases need more of them.

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