Math 6 Unit 1 worksheet: Greatest common factor by listing (answer key)

ANSWERS AT A GLANCE

1. (a) 7 (b) 6 (c) 8 (d) 9 (e) 1 (f) 19 (g) 25 (h) 18

2. (a) $\frac{2}{5}$ (b) $\frac{4}{7}$ (c) $\frac{5}{8}$

3. 11 bags, each with 5 glow sticks and 7 whistles.

4. 5 singers in each row, 7 rows in all.

5. (a) 48 cm, 6 patches. (b) 16 cm, 54 patches.

6. 24 and 36, or 36 and 48.

7. His list of factors of 36 is missing 12 (from 3 × 12); the GCF is 12.

WORKED SOLUTIONS

1. Find the greatest common factor by listing factors.

(a) 14 and 21 (b) 30 and 42 (c) 32 and 56 (d) 45 and 63

(e) 28 and 45 (f) 19 and 57 (g) 50 and 75 (h) 36, 54 and 90

ANSWER

(a) 7 (b) 6 (c) 8 (d) 9 (e) 1 (f) 19 (g) 25 (h) 18

SOLUTION

(a) Factors of 14: 1, 2, 7, 14. Going down, 14 does not divide 21 but 7 does (21 = 7 × 3). GCF(14, 21) = 7.

(b) Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30. 30 does not divide 42; 15 and 10 fail because 42 does not end in 0 or 5; 42 = 6 × 7. GCF(30, 42) = 6.

(c) Factors of 32: 1, 2, 4, 8, 16, 32. 56 ÷ 32 and 56 ÷ 16 leave remainders; 56 = 8 × 7. GCF(32, 56) = 8.

(d) Factors of 45: 1, 3, 5, 9, 15, 45. 45 and 15 do not divide 63 (63 does not end in 0 or 5); 63 = 9 × 7. GCF(45, 63) = 9.

(e) Factors of 28: 1, 2, 4, 7, 14, 28. 45 is odd, so 28, 14, 4 and 2 fail, and 45 ÷ 7 leaves 3. Only 1 is left: GCF(28, 45) = 1, so 28 and 45 are relatively prime.

(f) 19 is prime, and 57 = 19 × 3, so GCF(19, 57) = 19, the smaller number itself.

(g) Factors of 50: 1, 2, 5, 10, 25, 50. 50 does not divide 75; 75 = 25 × 3. GCF(50, 75) = 25.

(h) Work down the factors of 36, the smallest: 36, 18, 12, 9, 6, 4, 3, 2, 1. 36 does not divide 54; 54 = 18 × 3 and 90 = 18 × 5. GCF(36, 54, 90) = 18.

2. Simplify each fraction in one step by dividing the top and bottom by their GCF.

(a) $\frac{36}{90}$ (b) $\frac{56}{98}$ (c) $\frac{75}{120}$

ANSWER

(a) $\frac{2}{5}$ (b) $\frac{4}{7}$ (c) $\frac{5}{8}$

SOLUTION

(a) GCF(36, 90) = 18, since going down the factors of 36, 36 fails and 90 = 18 × 5. So $\frac{36}{90} = \frac{36 \div 18}{90 \div 18} = \frac{2}{5}$.

(b) GCF(56, 98) = 14: the factors of 56 above 14 are 56 and 28, and neither divides 98, but 98 = 14 × 7. So $\frac{56}{98} = \frac{4}{7}$.

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(c) GCF(75, 120) = 15: 75 and 25 do not divide 120, but 120 = 15 × 8. So $\frac{75}{120} = \frac{5}{8}$.

In each answer the top and bottom are relatively prime, so it cannot be simplified further.

3. Samir is filling party bags for a birthday. There are 55 glow sticks and 77 whistles. Every bag must hold exactly the same things, and nothing may be left over. What is the largest number of bags Samir can fill, and what goes in each bag?

ANSWER

11 bags, each with 5 glow sticks and 7 whistles.

SOLUTION

The glow sticks are shared equally among the bags with none left, so the number of bags is a factor of 55; the whistles make it a factor of 77 too. The most bags is the GCF.

Work down the factors of 55, the smaller number, until one divides 77. That happens first at 11: 55 = 11 × 5 and 77 = 11 × 7, and since 5 and 7 share no factor but 1, nothing larger works. So GCF(55, 77) = 11.

Samir fills 11 bags, each with 55 ÷ 11 = 5 glow sticks and 77 ÷ 11 = 7 whistles.

4. The school choir has 10 sopranos and 25 altos. On stage, each row holds singers of one part only, and every row has the same number of singers. What is the most singers a row can hold, and how many rows will there be?

ANSWER

5 singers in each row, 7 rows in all.

SOLUTION

The parts do not mix, so this time the GCF is the size of a row. The sopranos fill rows of equal length with nobody left out, so the row length is a factor of 10; the altos make it a factor of 25 too. The longest row is the GCF.

10 = 5 × 2 and 25 = 5 × 5, and 2 and 5 share no factor but 1, so GCF(10, 25) = 5.

Each row holds 5 singers. There are 2 soprano rows and 5 alto rows, 7 rows in all.

5. Yuna is covering a banner 96 cm wide and 144 cm long with identical square felt patches, edge to edge, with no gaps, no overlaps and no patch cut.

(a) What is the largest side length the patches can have, in whole centimeters, and how many patches does she need?

(b) Her patches must fit through a frame no more than 20 cm wide, so each side can be at most 20 cm. What is the largest side she can use now, and how many patches does that take?

ANSWER

(a) 48 cm, 6 patches. (b) 16 cm, 54 patches.

SOLUTION

(a) A row of patches along the 96 cm side fits exactly, so the side length is a factor of 96; the 144 cm side makes it a factor of 144 too. The largest side is the GCF. Work down the factors of 96: 96 does not divide 144, but 144 = 48 × 3, and 96 = 48 × 2. So GCF(96, 144) = 48. The banner is 2 patches by 3 patches, 6 patches.

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(b) The side must still be a common factor of 96 and 144. Every common factor is a factor of the GCF, 48: these are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. The largest that is at most 20 is 16. Then $96 \div 16 = 6$ and $144 \div 16 = 9$, so she needs $6 \times 9 = 54$ patches.

6. Two different whole numbers are each between 20 and 50, and their greatest common factor is 12. What could the two numbers be? Find every possible pair.

ANSWER

24 and 36, or 36 and 48.

SOLUTION

Each number has 12 as a factor, so each is a multiple of 12. Between 20 and 50 these are 24, 36 and 48, which make three pairs.

GCF(24, 36) = 12: 24 does not divide 36, and $36 = 12 \times 3$. This pair works.

GCF(24, 48) = 24, since $48 = 24 \times 2$. The GCF is too big, so this pair fails.

GCF(36, 48) = 12: 36 and 18 do not divide 48, and $48 = 12 \times 4$. This pair works.

7. Isaac finds GCF(36, 48). He writes the factors of 36 as 1, 2, 3, 4, 6, 9, 18, 36, picks out the ones that also divide 48, and says the GCF is 6. What has gone wrong, and what is the correct GCF?

ANSWER

His list of factors of 36 is missing 12 (from 3×12); the GCF is 12.

SOLUTION

Pairing finds every factor: $36 = 1 \times 36 = 2 \times 18 = 3 \times 12 = 4 \times 9 = 6 \times 6$. Isaac kept 3 but dropped its partner 12, so his list was not complete.

12 divides 48 ($48 = 12 \times 4$), and no larger factor of 36 does (36 and 18 both fail). So GCF(36, 48) = 12. His answer 6 is a common factor, but not the greatest one.

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