Math 6 Unit 1 worksheet: The sieve of Eratosthenes (answer key)

ANSWERS AT A GLANCE

1. (a) neither (b) prime (c) composite, $51 = 3 \times 17$ (d) composite, $35 = 5 \times 7$ (e) composite, $57 = 3 \times 19$ (f) prime (g) composite, $65 = 5 \times 13$ (h) prime
2. 7 rows of 29 students
3. (a) Only the columns of numbers ending in 1, 3, 7 and 9. (b) For example $21 = 3 \times 7$, put out in round 3.
4. (a) 210 is a multiple of each of them, so each leaves remainder 1 in 211. (b) 11 and 13; neither divides 211, so 211 is prime.
5. (a) Smallest 25, largest 48. (b) Smallest 169, largest 288.

WORKED SOLUTIONS

1. Decide whether each number is prime, composite or neither. For each composite number, write it as a product of two whole numbers greater than 1.

(a) 1 (b) 29 (c) 51 (d) 35

(e) 57 (f) 73 (g) 65 (h) 97

ANSWER

(a) neither (b) prime (c) composite, $51 = 3 \times 17$ (d) composite, $35 = 5 \times 7$ (e) composite, $57 = 3 \times 19$ (f) prime (g) composite, $65 = 5 \times 13$ (h) prime

SOLUTION

(a) 1 has only one factor, itself, so it is neither prime nor composite.

(b) $6 \times 6 = 36 > 29$, so only 2, 3 and 5 need testing. 29 is odd, $3 \times 9 = 27$ leaves remainder 2, and it does not end in 0 or 5. Prime.

(c) $51 = 3 \times 17$. Composite.

(d) $35 = 5 \times 7$. Composite.

(e) $57 = 3 \times 19$. Composite.

(f) $9 \times 9 = 81 > 73$, so test 2, 3, 5, 7. 73 is odd, $3 \times 24 = 72$ leaves remainder 1, it does not end in 0 or 5, and $7 \times 10 = 70$ leaves remainder 3. Prime.

(g) $65 = 5 \times 13$. Composite.

(h) $10 \times 10 = 100 > 97$, so test 2, 3, 5, 7. 97 is odd, $3 \times 32 = 96$ leaves remainder 1, it does not end in 0 or 5, and $7 \times 13 = 91$ leaves remainder 6. Prime.

2. The sixth grade has 203 students. For the class photo they will stand in equal rows, with more than one row and more than one student in each row. What is the fewest number of rows they can use, and how many students stand in each row?

ANSWER

7 rows of 29 students

SOLUTION

The number of rows must be a factor of 203 greater than 1, so the fewest rows is the smallest factor greater than 1. That factor is always a prime, so try the primes in order: 203 is odd, its digit sum 5 is not a multiple of 3, and it does not end in 0 or 5, so 2, 3 and 5 fail.

Next, $203 = 7 \times 29$, so the fewest rows is 7, with 29 students in each row.

3. Write the numbers 1 to 100 ten to a row, so the columns are the numbers ending in 1, in 2, and so on up to 0.

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(a) After the rounds for 2 and 5, which columns still hold lit numbers other than 2 and 5? Explain why every prime greater than 5 ends in 1, 3, 7 or 9.

(b) Lucas says every number ending in 1, 3, 7 or 9 is prime. Give a number that shows he is wrong, and name the round that puts it out.

ANSWER

(a) Only the columns of numbers ending in 1, 3, 7 and 9. (b) For example 21 = 3 × 7, put out in round 3.

SOLUTION

(a) Round 2 puts out every even number after 2: the columns ending in 2, 4, 6, 8 and 0 (all but 2 itself). Round 5 puts out every multiple of 5 after 5: the column ending in 5 (all but 5 itself). The columns left are those ending in 1, 3, 7 and 9.

A prime greater than 5 is not a multiple of 2 (so it does not end in 0, 2, 4, 6, 8) and not a multiple of 5 (so it does not end in 0 or 5). That leaves only 1, 3, 7 and 9 for its last digit.

(b) The last digit only rules out the factors 2 and 5. For example 21 = 3 × 7 ends in 1 and is put out in round 3. (Others: 27, 33, 39, 49, 51, 57, 63, 69, 77, 81, 87, 91, 93, 99.)

4. Ethan multiplies the first four primes and adds 1: 2 × 3 × 5 × 7 + 1 = 211.

(a) Explain why none of 2, 3, 5 and 7 divides 211, without dividing.

(b) Which other primes must Ethan test to decide whether 211 is prime? Test them, and decide.

ANSWER

(a) 210 is a multiple of each of them, so each leaves remainder 1 in 211. (b) 11 and 13; neither divides 211, so 211 is prime.

SOLUTION

(a) 210 = 2 × 3 × 5 × 7 is a multiple of 2, of 3, of 5 and of 7. The next multiple of each is more than 1 past 210, so 211, just 1 more, is not a multiple of any of them: each leaves remainder 1.

(b) 14 × 14 = 196 ≤ 211 and 15 × 15 = 225 > 211, so the primes to test are those up to 14:
2, 3, 5, 7, 11, 13. Part (a) has done the first four. 11 × 19 = 209 leaves remainder 2, and 13 × 16 = 208 leaves remainder 3. No prime up to 13 divides 211, so 211 is prime.

5. (a) Priya runs the sieve on a board from 1 to some number. Her board needs exactly the rounds for 2, 3 and 5, and no others. What is the smallest number her board could end at? The largest?

(b) Omar's board needs exactly six rounds. What is the smallest number his board could end at? The largest?

ANSWER

(a) Smallest 25, largest 48. (b) Smallest 169, largest 288.

SOLUTION

A board ending at N needs a round for each prime p with $p \times p \leq N$.

(a) Round 5 is needed, so $5 \times 5 = 25 \leq N$. Round 7 is not, so $7 \times 7 = 49 > N$. So N is from 25 to 48.

(b) The first six primes are 2, 3, 5, 7, 11, 13. Round 13 is needed, so $13 \times 13 = 169 \leq N$; round 17 is not, so $17 \times 17 = 289 > N$. So N is from 169 to 288.

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