Math 6 Unit 1 worksheet: Every number has exactly one prime factorization (answer key)

ANSWERS AT A GLANCE

1. (a) $2^2 \cdot 3 \cdot 5$ (b) $2^3 \cdot 3 \cdot 7^2$ (c) $2 \cdot 3^2 \cdot 5$ (d) $2^3 \cdot 3 \cdot 11$ (e) $3^2 \cdot 5 \cdot 7$ (f) $2^3 \cdot 5^3$
2. (a) Row 1: 1, 11. Row 2: 2, 22. Row 4: 4, 44. Row 8: 8, 88. (b) 1, 2, 4, 8, 11, 22, 44, 88: 8 factors (c) 6 are even
3. (a) $3^3 \cdot 7 = 189$ (b) $2 \cdot 3^3 \cdot 7 = 378$
4. (a) 3 zeros (b) 2 zeros
5. (a) 7 (b) 10
6. (a) $91 = 7 \cdot 13$ has only two primes, and three factors greater than 1 would need at least three. (b) 6, for example $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$.
7. (a) $36 = 6^2$ is on both lists (so is $1{,}296 = 6^4$). (b) No: every power of 6 has a 3 in its prime factorization and no power of 4 does.

WORKED SOLUTIONS

1. Each product is made of whole numbers greater than 1. Rewrite it as a prime factorization in tidy form (primes from smallest to largest, repeats gathered into powers). Do not multiply it out.

(a) $3 \cdot 2 \cdot 5 \cdot 2$ (b) $7 \cdot 2 \cdot 7 \cdot 3 \cdot 2 \cdot 2$ (c) $9 \cdot 2 \cdot 5$

(d) $4 \cdot 6 \cdot 11$ (e) $15 \cdot 21$ (f) $10 \cdot 10 \cdot 10$

ANSWER

(a) $2^2 \cdot 3 \cdot 5$ (b) $2^3 \cdot 3 \cdot 7^2$ (c) $2 \cdot 3^2 \cdot 5$ (d) $2^3 \cdot 3 \cdot 11$ (e) $3^2 \cdot 5 \cdot 7$ (f) $2^3 \cdot 5^3$

SOLUTION

(a) All four are already prime. Two 2s, one 3, one 5: $2^2 \cdot 3 \cdot 5$.

(b) All prime. Three 2s, one 3, two 7s: $2^3 \cdot 3 \cdot 7^2$.

(c) $9 = 3 \cdot 3$ is not prime, so split it: $2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$.

(d) $4 = 2 \cdot 2$ and $6 = 2 \cdot 3$: $2 \cdot 2 \cdot 2 \cdot 3 \cdot 11 = 2^3 \cdot 3 \cdot 11$.

(e) $15 = 3 \cdot 5$ and $21 = 3 \cdot 7$: $3 \cdot 3 \cdot 5 \cdot 7 = 3^2 \cdot 5 \cdot 7$.

(f) Each $10 = 2 \cdot 5$, so there are three 2s and three 5s: $2^3 \cdot 5^3$.

2. $88 = 2^3 \cdot 11$. Every factor of 88 is built from these primes.

(a) Make a table of the factors. Label the rows 1, 2, 4 and 8 (no 2s, one 2, and so on) and the columns 1 and 11. Fill each cell with its row label times its column label.

(b) List all the factors of 88 from smallest to largest. How many are there?

(c) How many of the factors are even?

ANSWER

(a) Row 1: 1, 11. Row 2: 2, 22. Row 4: 4, 44. Row 8: 8, 88. (b) 1, 2, 4, 8, 11, 22, 44, 88: 8 factors (c) 6 are even

SOLUTION

(a) Each cell uses some of the 2s (its row) and some of the 11s (its column): Row 1: 1, 11. Row 2: 2, 22. Row 4: 4, 44. Row 8: 8, 88.

(b) A number divides 88 exactly when its primes appear in $2^3 \cdot 11$ at least as many times, so the table holds every factor. No two cells are equal: by uniqueness, a different number of 2s or of 11s makes a different number. In order: 1, 2, 4, 8, 11, 22, 44, 88, which is 8 factors.

(c) Row 1 holds the factors with no 2, which are odd; every other cell has at least one 2, so it is even. That leaves 6 even factors.

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3. $n = 2^5 \cdot 3^3 \cdot 7$. Without multiplying it out, find

(a) the largest odd factor of n;

(b) the largest factor of n that is even but not a multiple of 4.

ANSWER

(a) $3^3 \cdot 7 = 189$ (b) $2 \cdot 3^3 \cdot 7 = 378$

SOLUTION

A factor of n can use at most five 2s, three 3s and one 7, and the more primes it uses, the larger it is.

(a) An odd factor has no 2s at all. The largest uses everything else: $3^3 \cdot 7 = 27 \cdot 7 = 189$.

(b) Even but not a multiple of 4 means exactly one 2. The largest such factor is $2 \cdot 3^3 \cdot 7 = 378$.

4. A number ends in a 0 when $10 = 2 \cdot 5$ divides it, in two 0s when $100 = 2^2 \cdot 5^2$ divides it, and so on.

(a) Without multiplying it out, how many zeros does $2^5 \cdot 3^2 \cdot 5^3$ end in?

(b) How many zeros does $8 \cdot 15 \cdot 35 \cdot 12$ end in?

ANSWER

(a) 3 zeros (b) 2 zeros

SOLUTION

(a) Ending in k zeros means $10^k = 2^k \cdot 5^k$ divides the number, so the number needs at least k 2s and at least k 5s. The factorization has five 2s and three 5s. So 10^3 divides it: $2^5 \cdot 3^2 \cdot 5^3 = 10^3 \cdot (2^2 \cdot 3^2) = 1000 \cdot 36$. But 10^4 would need four 5s, and by uniqueness the number has no other factorization with more, so 10^4 does not divide it. It ends in exactly 3 zeros (36,000).

(b) $8 = 2^3$, $15 = 3 \cdot 5$, $35 = 5 \cdot 7$ and $12 = 2^2 \cdot 3$, so the product is $2^5 \cdot 3^2 \cdot 5^2 \cdot 7$. There are only two 5s, so 10^2 divides it and 10^3 does not: exactly 2 zeros. **Check.** The product is $100 \cdot 504 = 50{,}400$.

5. Without multiplying out $34 \cdot 45 \cdot 91$, find the smallest whole number you could multiply it by so that the result is divisible by

(a) 98;

(b) 100.

ANSWER

(a) 7 (b) 10

SOLUTION

$34 = 2 \cdot 17$, $45 = 3^2 \cdot 5$ and $91 = 7 \cdot 13$, so the product is $2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13 \cdot 17$. Multiplying by a whole number k adds the primes of k to this list, and by uniqueness the result has no other factorization.

(a) $98 = 2 \cdot 7^2$ needs one 2 and two 7s. The product has the 2 but only one 7, so k must bring a 7. The smallest such k is 7.

(b) $100 = 2^2 \cdot 5^2$ needs two 2s and two 5s. The product has one of each, so k must bring a 2 and a 5: k is a multiple of 10, and the smallest is 10.

Check. The product is 139,230; $139{,}230 \cdot 7 = 974{,}610 = 98 \cdot 9{,}945$ and $139{,}230 \cdot 10 = 1{,}392{,}300 = 100 \cdot 13{,}923$.

6. (a) Isaac says he can write 91 as a product of three whole numbers, each greater than 1. Explain why he cannot.

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(b) What is the greatest number of whole numbers, each greater than 1, that multiply to 600? Give an example.

ANSWER

(a) 91 = 7 · 13 has only two primes, and three factors greater than 1 would need at least three. (b) 6, for example 2 · 2 · 2 · 3 · 5 · 5.

SOLUTION

(a) Each whole number greater than 1 has at least one prime in its factorization. If 91 were a product of three of them, splitting each into primes would give a prime factorization of 91 with at least three primes. But 91 = 7 · 13, two primes, and by uniqueness that is its only prime factorization. So Isaac cannot do it.

(b) $600 = 2^3 \cdot 3 \cdot 5^2$, which has 3 + 1 + 2 = 6 primes counting repeats. By the reasoning in (a), no product of more than 6 whole numbers greater than 1 can equal 600, and 2 · 2 · 2 · 3 · 5 · 5 = 600 uses exactly 6. The greatest number is 6.

7. Zoe lists the powers of 6: 6, 36, 216, … Ben lists the powers of 36: 36, 1,296, … Jin lists the powers of 4: 4, 16, 64, …

(a) Show that Zoe's list and Ben's list share a number.

(b) Will Zoe's list and Jin's list ever share a number, however far they go? Explain using prime factorizations.

ANSWER

(a) $36 = 6^2$ is on both lists (so is $1{,}296 = 6^4$). (b) No: every power of 6 has a 3 in its prime factorization and no power of 4 does.

SOLUTION

(a) 36 is the first number on Ben's list and 6 · 6 = 36 is the second on Zoe's. (Every power of 36 is a power of 6, since $36 = 6^2$.)

(b) A power of 4 is 2 · 2…2, a prime factorization with only 2s. A power of 6 is (2 · 3)(2 · 3)…(2 · 3), a prime factorization with at least one 3. If some number were on both lists, it would have one factorization with a 3 and one without, two different prime factorizations, which uniqueness rules out. So the lists never meet.

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