Dividing by multiplying by the reciprocal

A jug holds six liters and each bottle takes three quarters of a liter. How do you find how many bottles it fills, quickly, whatever the fractions are?

WORDS TO KNOW

Reciprocal: two numbers are reciprocals when their product is 1. The reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$, the fraction turned upside down.

Divisor: the number you divide by. In $\frac{2}{3} \div \frac{4}{7}$ it is $\frac{4}{7}$.

Simplest form: a fraction whose top and bottom have no common factor except 1.

Estimate: a quick guess at the size of the answer, made before you work it out.

KEY IDEAS

1. Find the reciprocal

Turn a fraction upside down. A whole number n is $\frac{n}{1}$, so its reciprocal is $\frac{1}{n}$. A number times its reciprocal is 1, so 0 has no reciprocal: 0 times any number is 0.

Number	$\frac{7}{9}$	**6**	$\frac{1}{20}$	**1**
Reciprocal	$\frac{9}{7}$	$\frac{1}{6}$	20	1

2. Keep, change, flip

Keep the first number as it is. Change $\div$ to $\times$, and in the same step flip the divisor. Then multiply top times top and bottom times bottom.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

A whole number goes over 1 first. Only the divisor flips.

3. Flip first, then simplify

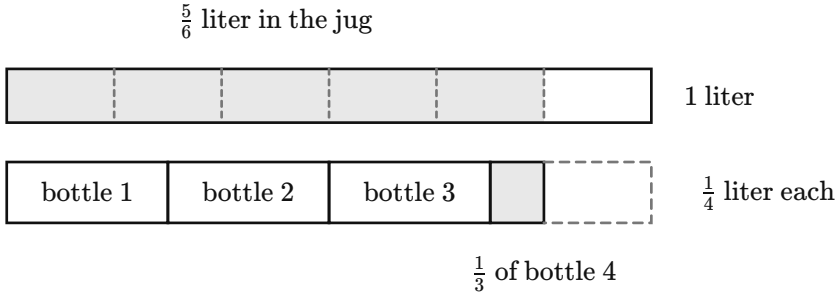
After the flip, divide out any factor that a top number shares with a bottom number. The numbers stay small.

4. Estimate, then check by multiplying

In the figure, $\frac{5}{6} \div \frac{1}{4}$ fills 3 bottles and a third. The divisor is less than 1, so the answer is more than $\frac{5}{6}$.

For a number a more than 0: dividing by a number between 0 and 1 gives more than a. Dividing by 1 gives a. Dividing by more than 1 gives less than a.

Check: answer $\times$ divisor $=$ the number you divided.



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WATCH OUT

- **Do not flip the first fraction, and do not swap the order.** Either slip gives the reciprocal of the right answer. The amount being divided goes first, and only the divisor flips.
- **Do not simplify across the ÷ sign.** The top of the first fraction and the bottom of the divisor both end up on top. Flip first, then divide out factors.
- **Do not just multiply.** Multiplying by a fraction is not dividing by it. You must flip the divisor. If the answer is too small or too big, the size rule will warn you.
- **The divisor cannot be 0.** But 0 divided by a fraction that is not 0 is 0.

WORKED EXAMPLES

Example 1. Find $\frac{4}{7} \div \frac{2}{5}$ in simplest form.

1. Keep $\frac{4}{7}$, change ÷ to ×, and flip $\frac{2}{5}$ to $\frac{5}{2}$.
2. Multiply across: $\frac{4}{7} \times \frac{5}{2} = \frac{20}{14}$.
3. Divide the top and bottom by 2: $\frac{20}{14} = \frac{10}{7}$.

Answer: $\frac{10}{7}$. It is more than 1 because $\frac{4}{7}$ is more than $\frac{2}{5}$. **Check:** $\frac{10}{7} \times \frac{2}{5} = \frac{20}{35} = \frac{4}{7}$.

Example 2. Find $9 \div \frac{6}{5}$ and $\frac{6}{7} \div 4$.

1. Write 9 as $\frac{9}{1}$ and flip $\frac{6}{5}$:
$$9 \div \frac{6}{5} = \frac{9}{1} \times \frac{5}{6} = \frac{45}{6} = \frac{15}{2}$$
2. The divisor is more than 1, so the answer should be less than 9. It is: $\frac{15}{2}$ is $7\frac{1}{2}$.
3. The reciprocal of 4 is $\frac{1}{4}$, so dividing by 4 takes a quarter:
$$\frac{6}{7} \div 4 = \frac{6}{7} \times \frac{1}{4} = \frac{6}{28} = \frac{3}{14}$$

Answer: $\frac{15}{2}$ and $\frac{3}{14}$. **Check:** $\frac{15}{2} \times \frac{6}{5} = \frac{90}{10} = 9$ and $\frac{3}{14} \times 4 = \frac{12}{14} = \frac{6}{7}$.

Example 3. Estimate $\frac{7}{12} \div \frac{14}{15}$, then find it and check.

1. Estimate: the divisor is less than 1, so the answer is more than $\frac{7}{12}$. And $\frac{7}{12}$ is less than $\frac{14}{15}$, so the answer is less than 1.
2. Flip first: $\frac{7}{12} \times \frac{15}{14}$. Now 7 goes into 7 and 14, and 3 goes into 15 and 12:
$$\frac{7 \times 15}{12 \times 14} = \frac{1 \times 5}{4 \times 2} = \frac{5}{8}$$
3. $\frac{5}{8}$ is between $\frac{7}{12}$ and 1, as the estimate said.

Answer: $\frac{5}{8}$. **Check:** $\frac{5}{8} \times \frac{14}{15} = \frac{70}{120} = \frac{7}{12}$.

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