Finding a missing value in equal ratios

Two ratios are meant to be equal, and you know three of their four numbers. What must the fourth one be?

WORDS TO KNOW

Equal ratios: two ratios where one is the other with both parts multiplied by the same number.

Across: reasoning from one ratio to the other, using the scaling number.

Scaling number: the number that takes each part of one ratio to the matching part of the other.

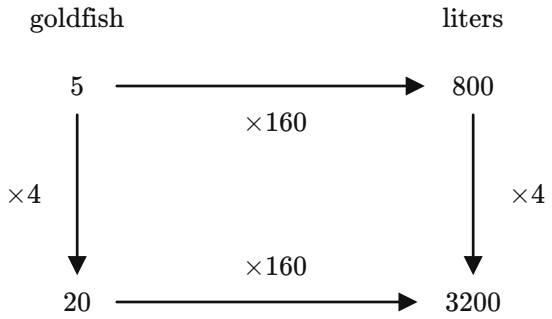
Within: reasoning inside one ratio, using how many times the second part is the first.

KEY IDEAS

1. Across: find the scaling number, then use it on the other part

A pond rule says 5 goldfish for every 800 liters. A pond of 3200 liters must keep the same comparison: $5 : 800 = ? : 3200$. The liters are known in both ratios, and $3200 \div 800 = 4$. The same scaling number must multiply the goldfish too: $5 \times 4 = 20$.

> Find the place where both numbers are known. Divide to get the scaling number. Multiply the other part by the same number.



2. Within: the inside number stays the same

Inside $5 : 800$ the liters are $800 \div 5 = 160$ times the goldfish. Scaling cannot change that, so in every equal ratio the liters are 160 times the goldfish.

> Across and within always give the same answer. Use whichever gives the friendlier number.

3. When neither number is whole, go through a friendlier ratio

For $4 : 7 = 10 : ?$, across is $10 \div 4 = 2.5$ and within is $7 \div 4 = 1.75$. Instead, 4 and 10 share the factor 2, so scale down by 2 first, then up by 5.

> $$4 : 7 = 2 : 3.5 = 10 : 17.5$$
>
> Divide both parts by a factor the two known numbers share. Then the step up is a whole number. Going all the way down to 1 part always works.

Check any answer by scaling: multiply both parts of the first ratio by the scaling number and see that both land.

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WATCH OUT

- **Line up the parts.** The first part of one ratio goes with the first part of the other. Mixing them answers a different question.
- **Multiply, do not add.** Ages grow by the same amount, not the same factor, so they do not stay in one ratio.
- **Units must match across.** If the rule is in kilograms and you have grams, change one first.
- **The answer may not fit the setting.** Equal ratios can call for 12.5 goldfish. You cannot keep half a goldfish, so the setting decides whether to round down or up.

WORKED EXAMPLES

Example 1. A choir has 11 sopranos for every 4 altos. A bigger choir keeps the same ratio and has 20 altos. How many sopranos does it have?

1. The question is $11 : 4 = ? : 20$.
2. The altos are known in both ratios: $20 \div 4 = 5$. The scaling number is 5.
3. Use the same number on the sopranos: $11 \times 5 = 55$.

Answer: 55 sopranos. **Check:** $11 \times 5 = 55$ and $4 \times 5 = 20$, so $11 : 4 = 55 : 20$.

Example 2. Find the missing value: $3 : 27 = 5 : ?$

1. Across, the scaling number would be $5 \div 3$, which is not whole.
2. Within, the second part is $27 \div 3 = 9$ times the first. That is a whole number, so use it.
3. In the new ratio the second part is also 9 times the first: $5 \times 9 = 45$.

Answer: 45. **Check:** 3 : 27 and 5 : 45 both have a second part 9 times the first.

Example 3. A juice stand fills 6 bottles of juice from every 20 oranges. How many bottles does it fill from 30 oranges?

1. Put the oranges first in both ratios: $20 : 6 = 30 : ?$
2. Across, $30 \div 20 = 1.5$ is not whole. Within, $20 \div 6$ and $6 \div 20$ are not whole either.
3. 20 and 30 are both even, so divide both parts of 20 : 6 by 2: $20 : 6 = 10 : 3$.
4. From 10 to 30 is $\times 3$, a whole number: $10 : 3 = 30 : 9$.

Answer: 9 bottles. **Check:** the across number 1.5 gives $20 \times 1.5 = 30$ and $6 \times 1.5 = 9$.

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