Distance on a number line

Two points on a number line are named by numbers, and one of the numbers may be negative. How far apart are the points?

WORDS TO KNOW

Coordinate: the number that names a point on the number line.

Distance: how many units apart two points are. It is a length, so it is never negative.

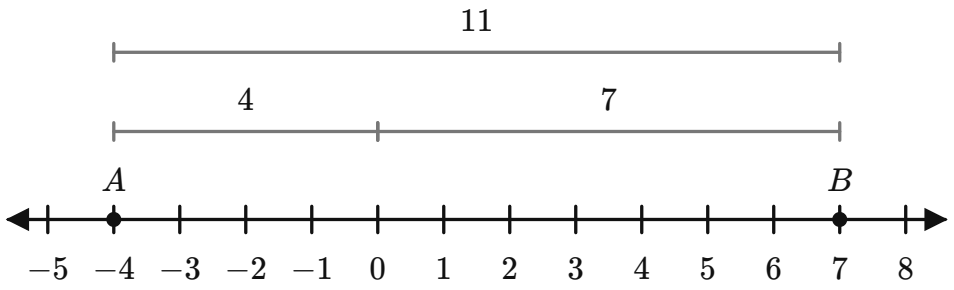
AB**:** the distance between points A and B, the length of the segment that joins them.

Absolute value: how far a number is from zero, written $|a|$. It drops the sign: $|-9| = 9$.

KEY IDEAS

1. Larger coordinate minus smaller coordinate

A number line works like a ruler. If both points are right of zero, subtract: from 2 to 9 is 9 − 2 = 7 units. Across zero, the gap splits at zero into two pieces, as in the picture, and subtraction still counts both. From −6 to 3 is 6 units to reach zero and 3 more, and 3 − (−6) = 3 + 6 = 9.



2. Absolute value means the order does not matter

Subtracting in the other order gives the opposite number: 5 − (−3) = 8, but −3 − 5 = −8. The size is the same, and the sign only tells a direction. A distance has no direction, so take the absolute value.

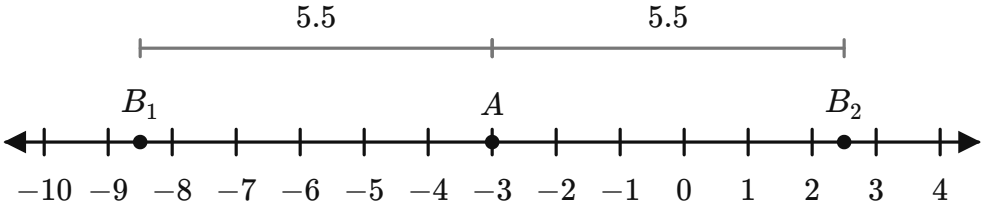
If A is at a and B is at b, then

$$AB = |b - a| = |a - b|$$

Subtract in either order, then take the absolute value.

3. A distance says how far, not which way

For any distance greater than 0, two points sit that far from a given point: one on each side. In the picture, 2.5 and −8.5 are both 5.5 units from −3.



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WATCH OUT

- **Do not just add the distances to zero.** That works only when the points are on opposite sides of zero. For −26 and −3, both left of zero, the distance is 26 − 3 = 23, not 26 + 3 = 29.
- **A distance is never negative.** If a subtraction gives −8, the distance is $|-8| = 8$.
- **Count units, not tick marks.** On a line marked every 10, the points at 10 and 40 are 30 units apart, not 3.

WORKED EXAMPLES

Example 1. Point C is at −10 and point D is at 7. Find CD.

1. Subtract the coordinates and take the absolute value.

$$CD = |7 - (-10)| = |7 + 10| = 17$$

2. The other order gives the same: $|-10 - 7| = |-17| = 17$.

Answer: $CD = 17$. **Check:** C is 10 units left of zero and D is 7 units right of it, and 10 + 7 = 17.

Example 2. Find the distance between the points at −6.2 and −1.7.

1. Subtract and take the absolute value.

$$|-1.7 - (-6.2)| = |-1.7 + 6.2| = |4.5| = 4.5$$

2. Both points are left of zero, so the distances to zero are subtracted, not added: 6.2 − 1.7 = 4.5.

Answer: the points are 4.5 units apart.

Example 3. Point A is at 1. Find every point that is 6.5 units from A.

1. One point is 6.5 units to the right: 1 + 6.5 = 7.5.
2. The other is 6.5 units to the left: 1 − 6.5 = −5.5.

Answer: 7.5 and −5.5. **Check:** $|7.5 - 1| = 6.5$ and $|-5.5 - 1| = |-6.5| = 6.5$.

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