Subtracting integers

How do you take away a number you do not have, such as subtracting −3 from 4? Every subtraction of integers can be turned into an addition you already know how to do.

WORDS TO KNOW

Opposite: the number the same distance from zero on the other side. The opposite of b is written $-b$, and the opposite of −5 is 5.

Zero pair: one positive counter (+1) and one negative counter (−1). Together they are worth 0.

Change: the ending value minus the starting value.

Difference: the answer to a subtraction. The difference $a - b$ can be positive, negative or zero.

Distance: how many units apart two numbers are on the number line. It is never negative.

KEY IDEAS

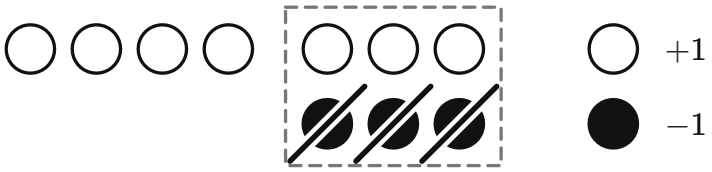
1. Subtracting is adding the opposite

To find 4 − (−3) with counters, you need three negatives to take away, but there are none. Put in three zero pairs: that adds 0, so the value is still 4. Now take the three negatives away. Their three positive partners stay behind, and seven positives are left. Taking away −3 did the same as adding 3.

For all integers a and b,

$$a - b = a + (-b) \qquad a - (-b) = a + b$$

Keep the first number. Change the subtraction to an addition. Replace the second number by its opposite.



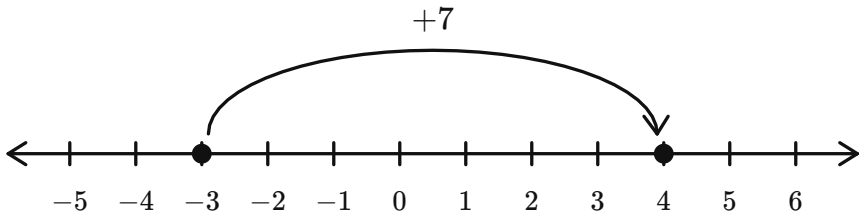
2. A difference answers "what do I add to get there?"

$a - b$ is the number you add to b to reach a. To get from −3 to 4 you move 7 to the right, so 4 − (−3) = 7. If the move goes right, the difference is positive; if it goes left, it is negative.

Swapping the order gives the opposite: $b - a = -(a - b)$.

The distance between a and b is $|a - b|$, in either order.

A change is ending minus starting. A positive change is a rise; a negative change is a fall.



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WATCH OUT

- **Change only the number being subtracted.** In −7 − 2 the −7 keeps its sign: −7 + (−2) = −9.
- **Do both moves.** Write 10 − (−6) as 10 + 6 = 16. Changing the minus sign to a plus but keeping −6 gives the wrong answer.
- **Order matters.** 3 − 9 = −6 but 9 − 3 = 6. In a string of subtractions, work from left to right, or first rewrite every subtraction as an addition.

WORKED EXAMPLES

Example 1. Find (a) 11 − 16, (b) −24 − (−10) and (c) 13 − (−8).

1. (a) Add the opposite: 11 − 16 = 11 + (−16). The signs differ, 16 − 11 = 5, and −16 is farther from zero, so the answer is −5.
2. (b) The opposite of −10 is 10: −24 − (−10) = −24 + 10. Then 24 − 10 = 14, and −24 is farther from zero, so the answer is −14.
3. (c) Subtracting a negative is adding a positive: 13 − (−8) = 13 + 8 = 21.

Answer: (a) −5 (b) −14 (c) 21. **Check:** add each answer back to the number subtracted. −5 + 16 = 11, −14 + (−10) = −24 and 21 + (−8) = 13.

Example 2. Simplify −5 − 9 + 4 − (−11) − 6.

1. Rewrite every subtraction as adding the opposite.
$$-5 + (-9) + 4 + 11 + (-6)$$
2. Now it is all addition, so you may reorder. Negatives: −5 + (−9) + (−6) = −20. Positives: 4 + 11 = 15.
3. Combine: −20 + 15 = −5.

Answer: −5. **Check:** left to right, −5 − 9 = −14, then −14 + 4 = −10, then −10 − (−11) = 1, then 1 − 6 = −5.

Example 3. A submarine rises from 140 meters below sea level to 35 meters below sea level. What is the change in its height?

1. Below sea level is negative, so it goes from −140 to −35.
2. Change is ending minus starting.
$$-35 - (-140) = -35 + 140 = 105$$
3. The change is positive, so it is a rise.

Answer: the submarine rose 105 meters. **Check:** −140 + 105 = −35, the ending height.

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