Math 7 Unit 1 test: Adding and subtracting integers (answer key)

12 questions, 37 points

ANSWERS AT A GLANCE

1. $n = -31$
2. 7
3. (a) Both sides equal −10. (b) $(-a) - (-b) = -a + b = b + (-a) = b - a$.
4. B is at −5, C is at −28, and $AC = 14$.
5. M is at 2.
6. (a) 103 degrees, a rise (b) 54 degrees larger (c) −56°F
7. (a) 18, a rise of 18 degrees. (b) The temperature climbs $|t|$ degrees up to zero, then the same $|t|$ degrees above it, so the rise is $|t| + |t|$.
8. (a) $a - b = -11$, $b - a = 11$ (b) 22 (c) $a - b$ and $b - a$ are opposites, and opposites lie on opposite sides of 0 at the same distance from it.
9. (a) 38 + (−76) + 24 + (−36) + (−13) = −63 (b) 2151 books
10. (a) 5,670 meters in Europe and 6,050 meters in Africa. (b) The difference is greater in Africa, by 380 meters.
11. (a) the Antarctic base; 25 degrees apart (b) −25°C; the Antarctic change was 18 degrees, a rise of 18 degrees
12. (a) 60, −75, 100 dollars (b) November to December (c) 85 dollars; 60 + (−75) + 100 = 85

WORKED SOLUTIONS

1. Find the integer n for which $-19 + n + 38 + (-12) = -24$. (2 points)

ANSWER

$n = -31$

SOLUTION

Addition can be done in any order, so first add the three known numbers: −19 + 38 + (−12) = 7. The equation becomes

$$7 + n = -24$$

So n is the move from 7 to −24 on the number line, which gives $n = -31$.

Check: −19 + (−31) + 38 + (−12) = −24.

SCORING

1 point for adding the known numbers to 7; 1 point for $n = -31$.

2. Find $-(-13) + (-|-18|) + |10 + (-22)|$. (3 points)

ANSWER

7

SOLUTION

Work out each of the three terms first.

- −(−13) is the opposite of −13, which is 13.
- $|-18| = 18$, so $-|-18| = -18$.
- 10 + (−22) = −12, since −22 is farther from zero, and $|-12| = 12$.

So the expression is 13 + (−18) + 12. Collect the positive numbers, 13 + 12 = 25, and then 25 + (−18) = 7.

SCORING

1 point for −(−13) = 13 and $-|-18| = -18$; 1 point for $|10 + (-22)| = 12$; 1 point for 7.

3. Ava says that $(-a) - (-b) = b - a$ for all integers a and b. (3 points)

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(a) Test her statement with $a = 7$ and $b = -3$.

(b) Show that she is right for all integers, giving a reason for each step.

ANSWER

(a) Both sides equal −10. (b) $(-a) - (-b) = -a + b = b + (-a) = b - a$.

SOLUTION

(a) With $a = 7$ and $b = -3$: $-a = -7$ and $-b = 3$, so the left side is −7 − 3 = −7 + (−3) = −10. The right side is −3 − 7 = −3 + (−7) = −10. Both sides equal −10.

(b) Subtracting a number is adding its opposite, and the opposite of $-b$ is b, so $(-a) - (-b) = -a + b$. Addition is commutative, so $-a + b = b + (-a)$. Adding the opposite of a is subtracting a, so $b + (-a) = b - a$. Together, $(-a) - (-b) = b - a$.

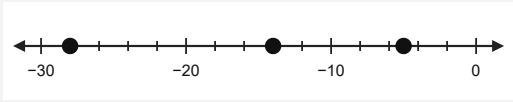
SCORING

1 point: (a) both sides worked out as −10. 1 point: (b) $(-a) - (-b)$ rewritten as $-a + b$, because subtracting is adding the opposite and $-(-b) = b$. 1 point: (b) reordered to $b + (-a)$ by the commutative property and read as $b - a$.

4. A token starts at point A, at −14 on a number line. It moves 9 units right to point B, then 23 units left to point C. Find the coordinates of B and C, and find the distance AC. (3 points)

ANSWER

B is at −5, C is at −28, and $AC = 14$.



SOLUTION

B: −14 + 9 = −5.

C: −5 + (−23) = −28.

$AC = |-28 - (-14)| = |-28 + 14| = |-14| = 14$.

Check: overall the token moved 23 − 9 = 14 units to the left.

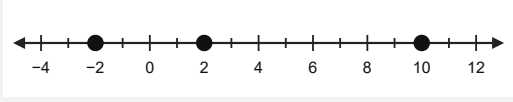
SCORING

1 point for B at −5; 1 point for C at −28; 1 point for $AC = 14$.

5. Point M lies between −2 and 10 on a number line, and M is twice as far from 10 as it is from −2. Find the coordinate of M, and check both distances. (3 points)

ANSWER

M is at 2.



SOLUTION

The two ends are $|10 - (-2)| = 12$ units apart. Since M is between them, its two distances add to 12. The distance to 10 is two parts and the distance to −2 is one part, so the 12 units are split into 3 equal parts of 12 ÷ 3 = 4 units: M is 4 units from −2 and 8 units from 10.

M is 4 units right of −2: −2 + 4 = 2.

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Check: $|2-(-2)|=4$ and $|10-2|=8$, and 8 is twice 4.

SCORING

1 point for the distance 12 between the ends; 1 point for splitting it as 4 and 8; 1 point for M at 2 with both distances checked.

6. Three record-breaking days in United States weather history: (3 points)

(a) On January 15, 1972, the temperature in Loma, Montana, went from −54°F to 49°F in one day. Find the change.

(b) On January 22, 1943, in Spearfish, South Dakota, it went from −4°F to 45°F in two minutes. How many degrees larger was the Loma rise than the Spearfish rise?

(c) On January 23, 1916, in Browning, Montana, the temperature fell 100 degrees in one day from 44°F. What was the temperature at the end of that fall?

ANSWER

(a) 103 degrees, a rise (b) 54 degrees larger (c) −56°F

SOLUTION

(a) Later minus earlier: 49 − (−54) = 49 + 54 = 103, a rise of 103 degrees.

(b) Spearfish: 45 − (−4) = 45 + 4 = 49, a rise of 49 degrees. The Loma rise was 103 − 49 = 54 degrees larger.

(c) A fall of 100 is a change of −100: 44 + (−100) = −56, so the low was −56°F.

SCORING

1 point for each part. In (b) the point needs both rises compared, not 49 − 45.

7. At 6 a.m. the temperature was −9°C. At 3 p.m. it was the opposite of the 6 a.m. temperature. (3 points)

(a) Find the change from 6 a.m. to 3 p.m.

(b) Liam says: "Whenever a temperature below zero rises to its opposite, the rise is twice the absolute value of the starting temperature." Explain why he is right.

ANSWER

(a) 18, a rise of 18 degrees. (b) The temperature climbs $|t|$ degrees up to zero, then the same $|t|$ degrees above it, so the rise is $|t|+|t|$.

SOLUTION

(a) The opposite of −9 is 9, so the 3 p.m. reading is 9°C. Later minus earlier: 9 − (−9) = 9 + 9 = 18, a rise of 18 degrees.

(b) Call the starting temperature t, with t below zero. It is $|t|$ degrees below zero, because absolute value is distance from zero. Its opposite is the same distance from zero on the other side, $|t|$ degrees above zero. On the thermometer the temperature climbs $|t|$ degrees to reach zero and then $|t|$ more, so the rise is $|t|+|t|$, twice the absolute value.

The subtraction says the same: the change is $-t-t$, and since t is negative, $-t=|t|$, so $-t-t=-t+(-t)=|t|+|t|$. In (a), $|-9|+|-9|=18$.

SCORING

(a) 1 point: 18 (rise). (b) 2 points: 1 for using that t and its opposite are each $|t|$ from zero on opposite sides; 1 for adding the climb to zero and the climb above zero to get $|t|+|t|$ (or the same through the subtraction).

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8. Let $a = -4$ and $b = 7$. (4 points)

(a) Evaluate $a - b$ and $b - a$.

(b) Find the distance between the point at $a - b$ and the point at $b - a$.

(c) Explain why, for any two different numbers a and b, the points at $a - b$ and $b - a$ are on opposite sides of 0, at the same distance from it.

ANSWER
(a) $a - b = -11$, $b - a = 11$ (b) 22 (c) $a - b$ and $b - a$ are opposites, and opposites lie on opposite sides of 0 at the same distance from it.

SOLUTION
(a) $a - b = -4 - 7 = -11$ and $b - a = 7 - (-4) = 7 + 4 = 11$.

(b) $|11 - (-11)| = |11 + 11| = 22$.

(c) $b - a$ is the opposite of $a - b$: adding them gives $(a - b) + (b - a) = 0$, a zero pair. Two opposites are the same distance from 0, since they have the same absolute value, and they have opposite signs, so one is left of 0 and the other right of it (neither is 0, because $a \neq b$).

SCORING
1 point for −11 and 11; 1 point for the distance 22; 2 points for explaining that the two differences are opposites (sum 0), so they have the same absolute value and opposite signs.

9. A school library counts books returned to its shelves as positive and books checked out as negative. One day, before lunch, 38 books were returned and 76 were checked out. After lunch, 24 were returned and 36 were checked out, and in the last hour 13 more were checked out. (3 points)

(a) Write the day's changes as a sum of integers, and find the net change in the number of books on the shelves.

(b) The shelves held 2214 books when the library opened. How many did they hold when it closed?

ANSWER
(a) 38 + (−76) + 24 + (−36) + (−13) = −63 (b) 2151 books

SOLUTION
(a) Returns are positive and checkouts negative:

$$38 + (-76) + 24 + (-36) + (-13)$$

Collect the positive numbers, 38 + 24 = 62, and the negative numbers, −76 + (−36) + (−13) = −125. Then 62 + (−125) = −63. The net change is −63 books.

(b) Add the net change to the starting count: 2214 + (−63) = 2151 books.

SCORING
1 point for a correct sum of five integers; 1 point for the net change −63; 1 point for 2151 books.

10. Heights are measured from sea level, and a height below sea level is negative. The highest point in Europe, at 5,642 meters, is Mount Elbrus; the lowest, at −28 meters, is the shore of the Caspian Sea. The highest point in Africa, at 5,895 meters, is Mount Kilimanjaro; the lowest, at −155 meters, is Lake Assal. (Heights are rounded to the nearest meter.) (3 points)

(a) For each continent, find the difference in height between its highest and lowest points.

(b) On which of the two continents is that difference greater, and by how many meters?

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ANSWER

(a) 5,670 meters in Europe and 6,050 meters in Africa. (b) The difference is greater in Africa, by 380 meters.

SOLUTION

(a) The difference is the highest height minus the lowest.

Mount Elbrus and the shore of the Caspian Sea: $5642 - (-28) = 5642 + 28 = 5670$ meters.

Mount Kilimanjaro and Lake Assal: $5895 - (-155) = 5895 + 155 = 6050$ meters.

(b) $6050 - 5670 = 380$, so the difference is greater in Africa, by 380 meters.

SCORING

1 point: the difference in Europe, 5,670 meters, found as highest minus a negative lowest. 1 point: the difference in Africa, 6,050 meters. 1 point: (b) greater in Africa, by 380 meters.

11. At noon one day, the thermometer at a research base in Greenland read −18°C and the one at a base in Antarctica read −43°C. (3 points)

(a) Which base was colder at noon, and how many degrees apart were the two readings?

(b) By evening the two thermometers showed the same reading. The Greenland temperature had fallen 7 degrees since noon. Find the evening reading, and the change in temperature at the Antarctic base from noon to evening.

ANSWER

(a) the Antarctic base; 25 degrees apart (b) −25°C; the Antarctic change was 18 degrees, a rise of 18 degrees

SOLUTION

(a) −43 is lower on the scale than −18, so the Antarctic base was colder. The distance between the readings is the higher minus the lower: $-18 - (-43) = -18 + 43 = 25$ degrees.

(b) The Greenland reading fell 7 degrees: $-18 + (-7) = -25$, so both thermometers read −25°C in the evening.

At the Antarctic base the change is later minus earlier: $-25 - (-43) = -25 + 43 = 18$. The change is positive, a rise of 18 degrees. Check: $-43 + 18 = -25$.

SCORING

(a) 1 point: the Antarctic base and the distance. (b) 1 point for the evening reading; 1 point for the Antarctic change as later minus earlier, with its value and "rise".

12. A school store records its profit each month. A loss is written as a negative profit. (4 points)

Month	September	October	November	December
Profit (dollars)	−25	35	−40	60

(a) Find the change in profit from each month to the next: September to October, October to November, and November to December.

(b) Between which two months did the profit rise the most?

(c) Find the change in profit from September to December, and show that it equals the sum of the three changes in (a).

ANSWER

(a) 60, −75, 100 dollars (b) November to December (c) 85 dollars; $60 + (-75) + 100 = 85$

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SOLUTION

(a) A change is the later value minus the earlier one.

September to October: $35 - (-25) = 35 + 25 = 60$ dollars, a rise.

October to November: $-40 - 35 = -40 + (-35) = -75$ dollars, a fall.

November to December: $60 - (-40) = 60 + 40 = 100$ dollars, a rise.

(b) The rises were 60 and 100 dollars, so the profit rose the most from November to December, by 100 dollars.

(c) September to December: $60 - (-25) = 60 + 25 = 85$ dollars. The three changes add to $60 + (-75) + 100 = 85$ dollars, the same, because each month's change starts where the one before it ended.

SCORING

2 points: (a) all three changes, each as later minus earlier with the subtraction of a negative done as an addition (60, −75, 100); 1 point if exactly one is wrong or has the wrong sign. 1 point: (b) November to December. 1 point: (c) 85 dollars, with the sum of the three changes shown to be 85.

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