Solving a proportion

A proportion with one number missing is an equation. Which number makes it true, and what is the quickest way to find it?

WORDS TO KNOW

Proportion: a statement that two ratios are equal, such as $\frac{6}{8} = \frac{x}{20}$.

Scale factor: the number that takes one ratio to the other, multiplying both of its parts.

Cross products: in $\frac{a}{b} = \frac{c}{d}$, the products ad and bc.

Solving a proportion: finding the missing number that makes the proportion true.

Unit rate: the amount of the top quantity for one of the bottom quantity, such as cups of stock per serving.

KEY IDEAS

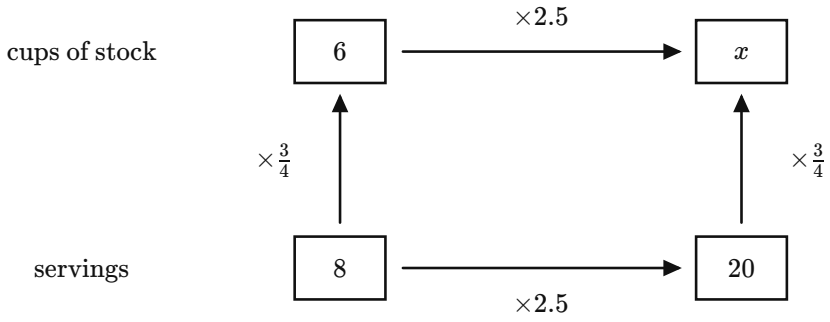
1. Two friendly moves: across or within

A soup for 8 people uses 6 cups of stock. For 20 people, solve $\frac{6}{8} = \frac{x}{20}$.

Across: from 8 servings to 20 is × 2.5, so the stock is 6 × 2.5 = 15 cups.

Within: each serving gets $\frac{6}{8} = \frac{3}{4}$ cup, so 20 servings get $20 \times \frac{3}{4} = 15$ cups.

Use whichever multiplier is easier with the numbers in front of you.



2. Two moves that always work

Unknown on top: multiply both sides by the number under the unknown.

$$\frac{x}{20} = \frac{6}{8} \quad \Rightarrow \quad x = 20 \cdot \frac{6}{8} = 15$$

Unknown underneath, or anywhere: set the cross products equal, then divide.

$$\frac{a}{b} = \frac{c}{d} \quad \text{exactly when} \quad ad = bc$$

For $\frac{6}{8} = \frac{27}{y}$ the cross products give $6y = 8 \cdot 27 = 216$, so $y = 36$.

3. Check twice

Put the answer back and test the cross products. Then ask whether its size makes sense: 27 cups is between 4 and 5 times 6 cups, so the servings should be between 32 and 40. And 36 is.

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WATCH OUT

- **Set both sides up the same way.** Keep the same quantity on top on both sides (stock over servings, and stock over servings), in the same units. A mixed-up proportion still has a solution, but to the wrong question.
- **Cross, do not multiply straight across.** In $\frac{a}{y} = \frac{c}{d}$, the cross products are ad and yc. Multiplying the two tops together is a common slip.
- **The answer need not be whole.** Three whole numbers can give a fraction or decimal answer. Keep it exact.
- **The situation decides the rounding.** People and muffins come whole. Flour can be a fraction of a cup.

WORKED EXAMPLES

Example 1. Solve (a) $\frac{x}{24} = \frac{7}{8}$ (b) $\frac{4}{15} = \frac{m}{25}$

1. (a) From 8 to 24 is a scale factor of 3, so $x = 7 \times 3 = 21$.
2. (b) No friendly factor takes 15 to 25, so multiply both sides by 25.

$$m = 25 \cdot \frac{4}{15} = \frac{100}{15} = \frac{20}{3} = 6\tfrac{2}{3}$$

Answer: (a) $x = 21$ (b) $m = 6\frac{2}{3}$. **Check:** (a) 21 · 8 = 168 = 24 · 7. (b) 4 · 25 = 100 and $15 \cdot \frac{20}{3} = 100$.

Example 2. Solve $\frac{14}{9} = \frac{35}{k}$

1. The unknown is underneath, so set the cross products equal: $14k = 9 \cdot 35 = 315$.
2. Divide both sides by 14: $k = \frac{315}{14} = 22.5$.
3. A scale factor confirms it: 35 is 2.5 times 14, so k is 2.5 times 9, which is 22.5.

Answer: $k = 22.5$. **Check:** 14 × 22.5 = 315.

Example 3. A muffin recipe uses 3 cups of flour for 16 muffins. How much flour is needed for 40 muffins? How many muffins can 10 cups of flour make?

1. Keep cups of flour on top and muffins underneath on both sides.
2. For 40 muffins: $\frac{3}{16} = \frac{f}{40}$. Multiply both sides by 40.

$$f = 40 \cdot \frac{3}{16} = \frac{120}{16} = 7.5$$

3. For 10 cups the unknown is a number of muffins, so it goes underneath: $\frac{3}{16} = \frac{10}{m}$. Cross products give $3m = 160$, so $m = 53\frac{1}{3}$.
4. Muffins come whole, and 10 cups is not quite enough for 54, so round down.

Answer: 7.5 cups of flour; 53 muffins. **Check:** 40 is 2.5 times 16, and 3 × 2.5 = 7.5.

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