Math 7 Unit 3 worksheet: Solving a proportion (answer key)

ANSWERS AT A GLANCE

1. (a) $x = 12$ (b) $n = 15$ (c) $b = 20$ (d) $c = 33$ (e) $w = 21$ (f) $m = 22.5$
2. (a) $p = 10.5$ (b) $d = 6$ (c) $r = -15$ (d) $q = \frac{3}{2}$ (e) $t = 1.5$
3. (a) 69 coins (b) 106 seconds
4. (a) 144 cm (b) 125 minutes
5. (a) No: $\frac{9}{13.5} = \frac{2}{3}$, but $\frac{6}{4} = \frac{3}{2}$. (b) He multiplied the two tops together and the two bottoms together. The cross products give $6y = 36$, so $y = 6$.
6. (a) $x = 12.6$ and $y = 12.6$. (b) They are equal: both cross product equations say 10 · (unknown) = 9 · 14 = 126. (c) $z = 12.6$, the same; $w = \frac{140}{9} = 15\frac{5}{9}$, not the same.
7. (a) 25 kg (b) 22.5 L (c) 30 bottles

WORKED SOLUTIONS

1. Solve each proportion. Check your answer with cross products.

(a) $\frac{x}{27} = \frac{4}{9}$ (b) $\frac{10}{16} = \frac{n}{24}$ (c) $\frac{36}{b} = \frac{9}{5}$ (d) $\frac{14}{21} = \frac{22}{c}$ (e) $\frac{18}{w} = \frac{24}{28}$ (f) $\frac{5}{12} = \frac{m}{54}$

ANSWER

(a) $x = 12$ (b) $n = 15$ (c) $b = 20$ (d) $c = 33$ (e) $w = 21$ (f) $m = 22.5$

SOLUTION

(a) From 9 to 27 is a scale factor of 3, so $x = 4 \times 3 = 12$. Check: 12 · 9 = 108 and 27 · 4 = 108.

(b) Simplify $\frac{10}{16} = \frac{5}{8}$, then multiply both sides by 24: $n = 24 \cdot \frac{5}{8} = 15$. Check: 10 · 24 = 240 and 16 · 15 = 240.

(c) Cross products: $9b = 36 \cdot 5 = 180$, so $b = 20$. Check: $\frac{36}{20} = 1.8$ and $\frac{9}{5} = 1.8$.

(d) Cross products: $14c = 21 \cdot 22 = 462$, so $c = 33$. Check: $\frac{14}{21} = \frac{2}{3}$ and $\frac{22}{33} = \frac{2}{3}$.

(e) Cross products: $24w = 18 \cdot 28 = 504$, so $w = 21$. Check: $\frac{18}{21} = \frac{6}{7}$ and $\frac{24}{28} = \frac{6}{7}$.

(f) Multiply both sides by 54: $m = 54 \cdot \frac{5}{12} = \frac{270}{12} = 22.5$. Check: 5 · 54 = 270 and 12 · 22.5 = 270.

2. Solve each proportion. Check your answer.

(a) $\frac{1.5}{0.4} = \frac{p}{2.8}$ (b) $\frac{-8}{d} = \frac{-12}{9}$ (c) $\frac{r}{35} = \frac{-6}{14}$ (d) $\frac{q}{\frac{2}{3}} = \frac{9}{4}$ (e) $\frac{4.8}{6} = \frac{1.2}{t}$

ANSWER

(a) $p = 10.5$ (b) $d = 6$ (c) $r = -15$ (d) $q = \frac{3}{2}$ (e) $t = 1.5$

SOLUTION

(a) Multiply both sides by 2.8: $p = 2.8 \cdot \frac{1.5}{0.4} = \frac{4.2}{0.4} = 10.5$. Check: 1.5 · 2.8 = 4.2 and 0.4 · 10.5 = 4.2.

(b) Cross products: $-12d = -8 \cdot 9 = -72$, so $d = 6$. Check: $\frac{-8}{6} = -\frac{4}{3}$ and $\frac{-12}{9} = -\frac{4}{3}$.

(c) Simplify $\frac{-6}{14} = \frac{-3}{7}$, then multiply both sides by 35: $r = 35 \cdot \frac{-3}{7} = -15$. Check: $\frac{-15}{35} = \frac{-3}{7}$.

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(d) Multiply both sides by $\frac{2}{3}$: $q = \frac{2}{3} \cdot \frac{9}{4} = \frac{18}{12} = \frac{3}{2}$. Check: $\frac{3}{2} \div \frac{2}{3} = \frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$.

(e) Cross products: $4.8t = 6 \times 1.2 = 7.2$, so $t = 1.5$. A scale factor agrees: 1.2 is $\frac{1}{4}$ of 4.8, so t is $\frac{1}{4}$ of 6, which is 1.5.

3. A coin press at a mint stamps 18 coins every 12 seconds, at a steady pace.

(a) How many coins does it stamp in 46 seconds?

(b) How many seconds does it take to stamp 159 coins?

ANSWER

(a) 69 coins (b) 106 seconds

SOLUTION

Keep coins on top and seconds underneath on both sides.

(a) $\frac{18}{12} = \frac{x}{46}$. Multiply both sides by 46: $x = 46 \cdot \frac{18}{12} = \frac{828}{12} = 69$ coins.

(b) The unknown is a time, so it goes underneath: $\frac{18}{12} = \frac{159}{t}$. Cross products: $18t = 12 \cdot 159 = 1908$, so $t = 106$ seconds.

Check (b): 18 · 106 = 1908 and 12 · 159 = 1908.

4. A weaving loom at a craft fair weaves 48 cm of cloth every 40 minutes, at a steady pace.

(a) How many centimeters of cloth does it weave in 2 hours?

(b) How many minutes does it take to weave a table runner 150 cm long?

ANSWER

(a) 144 cm (b) 125 minutes

SOLUTION

(a) The rate uses minutes, so change 2 hours to 120 minutes first. With centimeters on top and minutes underneath, $\frac{48}{40} = \frac{L}{120}$. Multiply both sides by 120: $L = 120 \cdot \frac{48}{40} = \frac{5760}{40} = 144$ cm.

(b) $\frac{48}{40} = \frac{150}{t}$. Cross products: $48t = 40 \cdot 150 = 6000$, so $t = 125$ minutes.

Check (b): 48 · 125 = 6000 and 40 · 150 = 6000.

5. Andre solved $\frac{9}{y} = \frac{6}{4}$. His work: $9 \cdot 6 = 4y$, so $54 = 4y$ and $y = 13.5$.

(a) Put $y = 13.5$ back into the proportion. Is the proportion true?

(b) Explain Andre's mistake, then solve the proportion correctly.

ANSWER

(a) No: $\frac{9}{13.5} = \frac{2}{3}$, but $\frac{6}{4} = \frac{3}{2}$. (b) He multiplied the two tops together and the two bottoms together. The cross products give $6y = 36$, so $y = 6$.

SOLUTION

(a) $\frac{9}{13.5} = \frac{2}{3}$, while $\frac{6}{4} = \frac{3}{2}$. The two ratios are not equal, so 13.5 is not a solution.

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(b) A cross product multiplies the top of one ratio by the bottom of the other. Andre multiplied the tops, $9 \cdot 6$, and the bottoms, $4 \cdot y$. The cross products are $9 \cdot 4$ and $6 \cdot y$:

$$6y = 9 \cdot 4 = 36, \quad \text{so} \quad y = 6$$

Check: $\frac{9}{6} = 1.5$ and $\frac{6}{4} = 1.5$.

6. (a) Solve $\frac{x}{14} = \frac{9}{10}$ and $\frac{9}{y} = \frac{10}{14}$.

(b) Compare your two answers. Use cross products to explain what you see.

(c) Decide whether each of these proportions has the same solution as the ones in (a), and solve it: $\frac{z}{9} = \frac{14}{10}$ $\frac{10}{w} = \frac{9}{14}$

ANSWER

(a) $x = 12.6$ and $y = 12.6$. (b) They are equal: both cross product equations say 10 · (unknown) = 9 · 14 = 126. (c) $z = 12.6$, the same; $w = \frac{140}{9} = 15\frac{5}{9}$, not the same.

SOLUTION

(a) Multiply both sides of the first by 14: $x = 14 \cdot \frac{9}{10} = \frac{126}{10} = 12.6$. For the second, the cross products give $10y = 9 \cdot 14 = 126$, so $y = 12.6$.

(b) The answers are equal. In both proportions the cross products pair the unknown with 10, and pair 9 with 14. So both become the same equation, 10 · (unknown) = 126, and an equation has the same solution whatever the unknown is called.

(c) In $\frac{z}{9} = \frac{14}{10}$ the cross products are $10z = 9 \cdot 14 = 126$, the same equation again, so $z = 12.6$.
In $\frac{10}{w} = \frac{9}{14}$ the unknown is paired with 9 and 10 with 14: $9w = 140$, so $w = \frac{140}{9} = 15\frac{5}{9}$, a different answer.

7. At a small olive mill, every 5 kg of olives pressed gives 0.9 L of olive oil.

(a) How many kilograms of olives are needed for 4.5 L of oil?

(b) Isabel's family brings the olives from three trees: 38 kg, 41 kg and 46 kg. How much oil should they expect?

(c) The oil is poured into bottles that hold 0.75 L each. How many bottles can they fill?

ANSWER

(a) 25 kg (b) 22.5 L (c) 30 bottles

SOLUTION

(a) With olives on top and oil underneath, $\frac{5}{0.9} = \frac{o}{4.5}$. From 0.9 L to 4.5 L is a scale factor of 5, so $o = 5 \times 5 = 25$ kg. Check: 5 · 4.5 = 22.5 and 0.9 · 25 = 22.5.

(b) The three trees give 38 + 41 + 46 = 125 kg of olives. With oil on top, $\frac{0.9}{5} = \frac{L}{125}$. From 5 kg to 125 kg is a scale factor of 25, so $L = 0.9 \times 25 = 22.5$ L.

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(c) One bottle holds 0.75 L, so $\frac{1}{0.75} = \frac{n}{22.5}$. Multiply both sides by 22.5: $n = \frac{22.5}{0.75} = 30$ bottles, exactly full.

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