Two-step equations with negatives, fractions and decimals

A number was multiplied, then something was added, and only the result is known. How do you get back to the number?

WORDS TO KNOW

Two-step equation: an equation of the form $px + q = r$, where the unknown appears once and p is not 0.

Constant term: the number added on its own. In $6 - 10k$ it is 6.

Solution: the number that makes the equation true when you put it in.

Coefficient: the number that multiplies the unknown. In $6 - 10k$ it is −10.

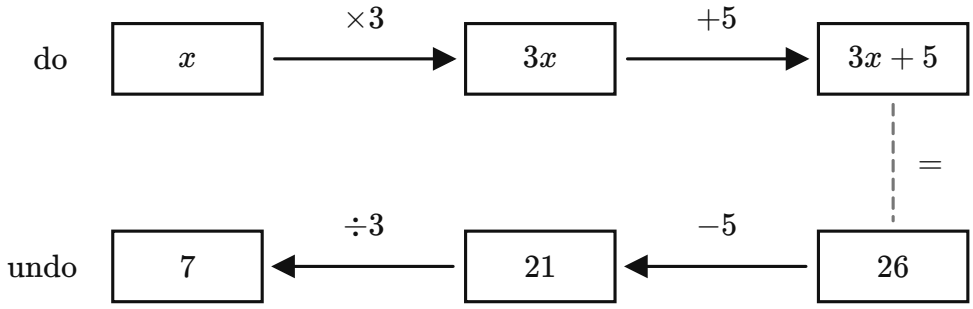
Reciprocal: the number you multiply by to get 1. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

KEY IDEAS

1. Undo the steps in reverse order

Read $3x + 5 = 26$ as a story about x: first it was multiplied by 3, then 5 was added. To get back to x, undo the last step first. Whatever you do, do it to both sides, so the two sides stay equal.

> For $px + q = r$: first subtract q from both sides, then divide both sides by p.
>
> Then check: put your answer back into the original equation.



2. Read the signs first

A minus sign belongs to the term after it. So $6 - 10k$ is $6 + (-10k)$: the coefficient is −10 and the constant is 6. An unknown written $-x$ has coefficient −1. Dividing both sides by a negative number is fine in an equation; the signs follow the rules for dividing signed numbers.

> A fraction coefficient is undone by its reciprocal: to undo $\frac{4}{5}x$, multiply both sides by $\frac{5}{4}$.
>
> Or clear fractions or decimals first: multiply every term on both sides by a common denominator, or by 10 or 100.

3. Equations from stories

Name the unknown with its unit. A start of q that changes by p each step ends at $px + q$ after x steps. If the amount goes down, p is negative. Check the answer in the story's own words, and ask if it can happen: a count must be a whole number, and a time cannot be negative.

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WATCH OUT

- **The coefficient keeps its sign.** In $6 - 10k = 46$ you divide by −10, not by 10. Dividing by 10 gives the wrong sign in the answer.
- **A move reaches every term.** If you divide first, or clear fractions, every term on both sides is divided or multiplied, not just the one with the unknown.
- **The reciprocal of a negative is negative.** To undo $-\frac{2}{5}p$, multiply by $-\frac{5}{2}$, since the product must be $+1$.
- **A negative time means a wrong sign.** If a story gives a negative number of minutes, look again at whether the amount went up or down.

WORKED EXAMPLES

Example 1. Solve $-12y + 7 = 151$.

1. The unknown was multiplied by −12, then 7 was added. Undo the adding first: subtract 7 from both sides, which gives $-12y = 144$.
2. Undo the multiplying: divide both sides by −12. A positive divided by a negative is negative, so $y = -12$.

Answer: $y = -12$. **Check:** −12(−12) + 7 = 144 + 7 = 151.

Example 2. Solve $\frac{3}{7}x - 4 = 5$.

1. Undo the subtracting first: add 4 to both sides, which gives $\frac{3}{7}x = 9$.
2. Undo the multiplying by $\frac{3}{7}$: multiply both sides by its reciprocal, $\frac{7}{3}$.

$$x = \frac{7}{3} \cdot 9 = 21$$

Answer: $x = 21$. **Check:** $\frac{3}{7} \cdot 21 = 9$, and 9 − 4 = 5.

Example 3. Heights on a canyon trail are measured from the rim, so points below it are negative. Wilhelm starts 40 m below the rim and walks down 12 m each minute. After how many minutes is he at a ledge 220 m below the rim?

1. Let t be the number of minutes. His height starts at −40 and changes by −12 each minute, so after t minutes it is $-40 - 12t$. The ledge is at −220.

$$-40 - 12t = -220$$

2. Add 40 to both sides: $-12t = -180$.
3. Divide both sides by −12: $t = 15$.

Answer: 15 minutes. **Check:** in 15 minutes he goes down 15 × 12 = 180 m, and 40 + 180 = 220 m below the rim.

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