Area of a circle

A circle has no straight sides to multiply. How much flat surface is inside one?

WORDS TO KNOW

Area: the amount of flat surface inside a shape, measured in square units such as cm^2 or m^2.

r^2: the radius squared, $r \times r$.

Exact answer: an area with π kept as a symbol, such as 10π m^2.

Rounded answer: an area with π replaced by a decimal, written with $\approx$ ("is about equal to").

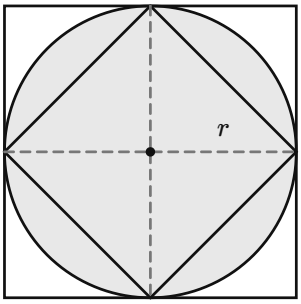
Square root ($\sqrt{\ }$): for a positive number, the positive number whose square it is. $\sqrt{400} = 20$, because $20 \times 20 = 400$.

Ring: the band between two circles with the same center.

KEY IDEAS

1. Area is pi times the radius squared

A square on one radius, r by r, has area r^2. The square around the circle holds four of these, and the diamond inside the circle holds two. The circle holds a little more than three: exactly π of them.



$$A = \pi r^2$$

Square the radius first, then multiply by π. With a diameter, halve it first, or use

$$A = \frac{\pi d^2}{4}$$

A radius in meters gives an area in square meters.

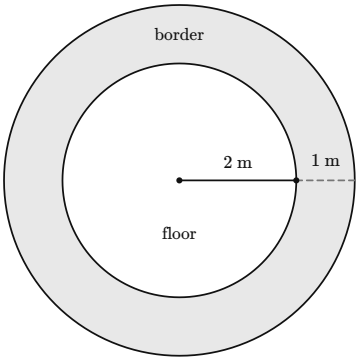
2. The radius from the area

When the area is known, the same formula works backwards. Divide by π, then take the square root. A radius is a length, so it is never negative.

$$r^2 = \frac{A}{\pi} \qquad r = \sqrt{\frac{A}{\pi}}$$

3. Pieces, rings and growing circles

A half circle has half the area of the full circle, and a quarter circle has a quarter. A ring is the big circle with the small one taken out.



Half circle: $\frac{1}{2}\pi r^2$. Quarter circle: $\frac{1}{4}\pi r^2$.

Ring between radii R and r: $\pi R^2 - \pi r^2$.

Multiply the radius by k and the area is multiplied by k^2. Twice the radius gives four times the area.

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WATCH OUT

- **Use the radius, not the diameter.** Putting d into πr^2 gives four times the true area. Halve the diameter first.
- **Area is not circumference.** The distance around the edge, $2\pi r$, is a length in cm. The surface inside, πr^2, is an area in square cm.
- **A ring is a subtraction.** Its area is $\pi R^2 - \pi r^2$, not the area of a circle whose radius is the width $R - r$.
- **Round only at the end.** Keep π as a symbol, and double the unrounded radius when you need a diameter.

WORKED EXAMPLES

Example 1. A round rug is 2.2 m across. Find its area, exactly and to one decimal place.

1. "Across" is the diameter, so the radius is half of it: $r = 2.2 \div 2 = 1.1$ m.
2. Square the radius, then multiply by π.

$$A = \pi \times 1.1^2 = 1.21\pi \approx 3.8$$

Answer: 1.21π m^2, about 3.8 m^2. **Check:** the rug fits in a square 2.2 m wide, area 4.84 m^2, and fills about three quarters of it: $0.75 \times 4.84 \approx 3.6$. The answer is a little more, as it should be.

Example 2. (a) A round lawn has area 625π m^2. Find its radius. (b) A round pond covers 300 m^2. Find its radius to one decimal place.

1. (a) Set the formula equal to the area: $\pi r^2 = 625\pi$. Divide both sides by π: $r^2 = 625$.
2. $25 \times 25 = 625$, so $r = 25$ m.
3. (b) Here no π cancels: $\pi r^2 = 300$, so

$$r^2 = \frac{300}{\pi} \approx 95.49$$

4. $9^2 = 81$ is too small and $10^2 = 100$ is too big, so r is between 9 and 10. The square root key gives $r \approx 9.77$.

Answer: (a) 25 m. (b) about 9.8 m. **Check:** $\pi \times 25^2 = 625\pi$, and $9.77^2 \approx 95.45$, very close to 95.49.

Example 3. A round pool has radius 4 m. A path 2 m wide runs all the way around it. Find the area of the path.

1. The pool and the path together make a big circle of radius $4 + 2 = 6$ m.
2. Take the pool out of the big circle.

$$\pi \times 6^2 - \pi \times 4^2 = 36\pi - 16\pi = 20\pi \approx 62.8$$

Answer: 20π m^2, about 62.8 m^2. **Check:** the pool alone is 16π m^2, so the path, though only 2 m wide, covers more ground than the pool.

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