Comparing numbers in scientific notation

Given two numbers in scientific notation, how can you tell at a glance which is larger, and about how many times larger it is?

WORDS TO KNOW

Scientific notation: $a \times 10^n$ with $1 \le a < 10$ and n an integer.

Exponent: the n on the 10.

How many times larger: the larger number divided by the smaller.

Leading part: the a in front.

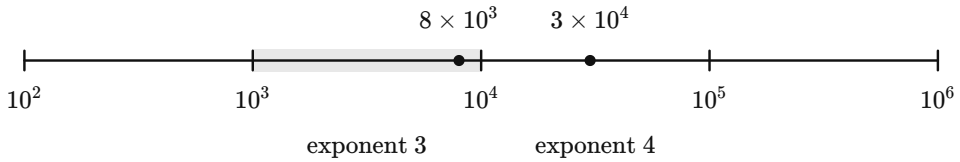
Order of magnitude: the power of ten 10^n in a number's scientific notation, its size to the nearest factor of ten.

KEY IDEAS

1. Check the exponent first

Because $1 \le a < 10$, a number with exponent n is at least 10^n and less than 10^{n+1}. So every number with exponent 4 sits above every number with exponent 3. The stretches never overlap.

> Of two positive numbers in scientific notation, the one with the larger exponent is larger, whatever the leading parts are.
>
> Negative exponents follow the same rule: $-3 > -5$, so any number with exponent -3 beats any number with exponent -5.



2. Same exponent: compare the leading parts

If the exponents match, compare the leading parts like decimals, place by place from the left. To put a list in order, sort by exponent first, then sort each group that shares an exponent by its leading parts.

3. How many times larger: divide

Divide the leading parts and subtract the exponents.

$$\frac{b \times 10^m}{a \times 10^n} = \frac{b}{a} \times 10^{m-n}$$

Before you divide: if the exponents differ by d, the larger number is more than 10^{d-1} times and less than 10^{d+1} times the smaller. If the larger number also has the smaller leading part, the ratio is below 10^d.

4. Estimate with one digit

Round each leading part to one digit, like $1.65 \times 10^8 \approx 2 \times 10^8$, then compare or divide in your head. If a leading part rounds up to 10, move a ten into the exponent.

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WATCH OUT

- **The rule needs true scientific notation.** 0.6×10^7 has the bigger exponent but is less than 9×10^6. Rewrite each number in the form first.
- **Negative exponents: compare them as integers.** $10^{-3} = 0.001$ is larger than $10^{-5} = 0.00001$, because $-3 > -5$.
- **The rule is for positive numbers.** For negative numbers the order flips: -5×10^4 is less than -3×10^2, because it is further below zero.

WORKED EXAMPLES

Example 1. Which is larger? (a) 4.2×10^7 or 9.8×10^6. (b) 2×10^{-3} or 8×10^{-5}.

1. In (a) the exponents are 7 and 6. Since $7 > 6$, 4.2×10^7 is larger, even though $9.8 > 4.2$.
2. In (b) the exponents are -3 and -5. On the number line -3 is to the right of -5, so 2×10^{-3} is larger, even though $8 > 2$.

Answer: (a) 4.2×10^7. (b) 2×10^{-3}. **Check:** written out, 42,000,000 > 9,800,000 and 0.002 > 0.00008.

Example 2. Order from least to greatest: 0.45×10^6, 48×10^4, 5.1×10^5, 490,000.

1. Only 5.1×10^5 is in the form, so rewrite the rest first.
2. $0.45 \times 10^6 = 4.5 \times 10^5$, $48 \times 10^4 = 4.8 \times 10^5$, $490{,}000 = 4.9 \times 10^5$.
3. All four now have exponent 5, so the leading parts decide: $4.5 < 4.8 < 4.9 < 5.1$.

Answer: $0.45 \times 10^6 < 48 \times 10^4 < 490{,}000 < 5.1 \times 10^5$. **Check:** as plain numbers, 450,000 < 480,000 < 490,000 < 510,000.

Example 3. The Earth's mass is about 6×10^{24} kg and the Moon's about 7.3×10^{22} kg. About how many times as heavy is the Earth?

1. Estimate first. The exponents differ by 2, and the Earth's leading part, 6, is the smaller one. So the ratio is between 10 and 100.
2. Divide the leading parts and subtract the exponents:

$$\frac{6 \times 10^{24}}{7.3 \times 10^{22}} = \frac{6}{7.3} \times 10^2 \approx 0.82 \times 100 = 82$$

Answer: about 80 times as heavy. The masses were rounded, so a round answer is all the data supports. **Check:** 82 is between 10 and 100, as step 1 said.

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