Multiplying, dividing and powering monomials

When you multiply, divide or raise single terms like 3x squared to a power, how do you get one number times powers of each letter, and what happens to the exponents?

WORDS TO KNOW

Monomial: a single term, a number times letters with positive whole-number exponents, like $-5a^4b^2$ or 7.

No exponent written: x means x^1, so a letter with no exponent has exponent one.

Simplified: each letter appears once, no zero or negative exponents, and the coefficient is in lowest terms.

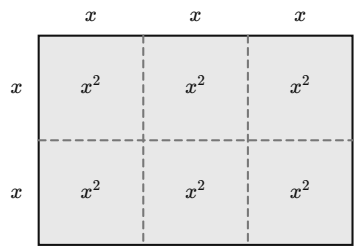
Coefficient: the number in front. In x it is 1, and in $-x^2$ it is −1.

Negative exponent: a^{-n} means $\frac{1}{a^n}$, a reciprocal, not a negative number.

KEY IDEAS

1. Multiplying: multiply the numbers, add the exponents

A monomial is a product, and you may reorder and regroup factors. So put the numbers together and each letter together. A rectangle $3x$ by $2x$ splits into $3 \cdot 2 = 6$ squares, each x^2, so $(3x)(2x) = 6x^2$.



Multiply the coefficients, with their signs. For each letter, add its exponents:

$$a^m \cdot a^n = a^{m+n}$$

A letter in only one factor passes into the answer unchanged. Different letters never combine.

2. Dividing: divide the numbers, subtract the exponents

Each factor on the bottom cancels one equal factor on top. Every letter on the bottom must be nonzero.

Divide the coefficients as a fraction in lowest terms. For each letter, subtract the smaller exponent from the larger. What is left stays on the side that had more:

$$\frac{x^5}{x^2} = x^3 \qquad \frac{x^2}{x^5} = \frac{1}{x^3} \qquad \frac{x^4}{x^4} = 1$$

3. A power of a monomial: the outside exponent reaches every factor

$(ab)^3 = (ab)(ab)(ab) = a^3b^3$, and $(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{12}$.

Raise the coefficient to the power, sign and all. Multiply each letter's exponent by the outside exponent:

$$(-3xy^4)^2 = (-3)^2x^2(y^4)^2 = 9x^2y^8$$

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4. Zero and negative exponents let a factor cross the fraction bar

For a nonzero base, $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$. With these, every rule above works for any whole-number exponent, positive, negative or zero. A factor with a negative exponent can move across the fraction bar, and its exponent changes sign.

$$\frac{a^{-2}b^3}{c^{-4}} = \frac{b^3c^4}{a^2}$$

Only factors move, never a term of a sum. And only the base the exponent belongs to moves: $3x^{-2} = \dfrac{3}{x^2}$, but $(3x)^{-2} = \dfrac{1}{9x^2}$.

WATCH OUT

- **Adding is not multiplying.** $3x^2 + 4x^2 = 7x^2$, but $(3x^2)(4x^2) = 12x^4$. A sum like $x^2 + x^3$ stays as two terms.
- **Raise the coefficient too.** $(3x)^2 = 9x^2$, not $3x^2$. But in $2x^3$ only the x is cubed, and in $-(2x)^2 = -4x^2$ the minus sign is outside, so it is not squared.
- **A negative exponent is not a negative number.** $4^{-2} = \dfrac{1}{16}$, not −16.

WORKED EXAMPLES

Example 1. Multiply $(3a^2b^5)(-4ab^3c)$.

1. Coefficients: 3 · (−4) = −12.
2. For a: the second factor has a^1, so 2 + 1 = 3. For b: 5 + 3 = 8.
3. The letter c is only in the second factor, so it passes through.

Answer: $-12a^3b^8c$. **Check:** at $a = 1$, $b = 2$, $c = 3$, the left side is (3 · 32)(−4 · 8 · 3) = 96 · (−96) = −9216, and the right side is −12 · 256 · 3 = −9216.

Example 2. Divide $\dfrac{-12m^7n^2}{30m^3n^6}$. The letters are nonzero.

1. Coefficients: $\dfrac{-12}{30} = -\dfrac{2}{5}$. For m the top has more: 7 − 3 = 4, so m^4 stays on top.
2. For n the bottom has more: 6 − 2 = 4, so n^4 stays on the bottom.

Answer: $-\dfrac{2m^4}{5n^4}$. **Check:** at $m = 1$, $n = 2$, the left side is $\dfrac{-48}{1920} = -\dfrac{1}{40}$, and the right side is $-\dfrac{2}{80} = -\dfrac{1}{40}$.

Example 3. Simplify $\dfrac{(3a^2b)^2}{6a^{-1}b^5}$ with positive exponents. The letters are nonzero.

1. Power first. Square every factor in the parentheses: $(3a^2b)^2 = 9a^4b^2$.
2. Divide the coefficients: $\dfrac{9}{6} = \dfrac{3}{2}$.
3. Subtract exponents. For a: 4 − (−1) = 5. For b: 2 − 5 = −3.
4. So the result is $\dfrac{3}{2}a^5b^{-3}$. Move b^{-3} down as b^3.

Answer: $\dfrac{3a^5}{2b^3}$. **Check:** at $a = 2$, $b = 2$, the top is $24^2 = 576$ and the bottom is $6 \cdot \dfrac{1}{2} \cdot 32 = 96$, so the left side is 6. The right side is $\dfrac{3 \cdot 32}{2 \cdot 8} = 6$.

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