Powers of powers, products and quotients

When a power, a product or a fraction is itself raised to a power, how many factors do you get, and of what?

WORDS TO KNOW

Power: a^n is n factors of a multiplied together. a is the base, n the exponent.

Power of a product: a product raised to a power, such as $(2x)^3$.

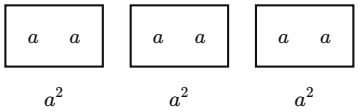
What an exponent touches: only the one thing just before it. In $2x^3$ only x is cubed; in $(2x)^3$ everything in the parentheses is.

Power of a power: a power raised to a power, such as $(x^2)^3$.

Power of a quotient: a fraction raised to a power, such as $\left(\frac{x}{2}\right)^3$.

KEY IDEAS

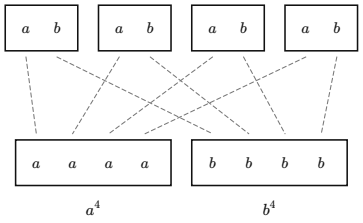
1. A power of a power: multiply the exponents



$(a^2)^3$ is three copies of a^2, and each copy is two factors of a. Three groups of two make six factors, so $(a^2)^3 = a^6$.

> **Power of a power.** $(a^m)^n = a^{mn}$
>
> Compare: $a^m \cdot a^n = a^{m+n}$. Two different groups lined up: add. One group repeated: multiply.

2. A power of a product: every factor gets the exponent



$(ab)^4$ is four copies of ab. The order of factors does not matter, so sort them: four a's and four b's.

> **Power of a product.** $(ab)^n = a^n b^n$, and $(abc)^n = a^n b^n c^n$.
>
> Read backward, it joins different bases with the same exponent: $a^n b^n = (ab)^n$.

3. A power of a quotient: the top and the bottom both get it

Multiplying fractions multiplies tops together and bottoms together. So three copies of $\frac{a}{b}$ give three a's on top and three b's below.

> **Power of a quotient.** For $b \neq 0$: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

WATCH OUT

- **Multiply, do not add.** $(x^4)^5 = x^{20}$, not x^9. Adding is for $x^4 \cdot x^5$.
- **The number in front gets the exponent too.** $(3y^4)^2 = 9y^8$, not $3y^8$. So does a minus sign: an even power of a negative number is positive, an odd power is negative.
- **Stacked exponents are different.** $(7^2)^3 = 7^6$, but 7^{2^3} means $7^{(2^3)} = 7^8$. Work a stack from the top down.
- **Never through a sum.** $(4+1)^2 = 25$, but $4^2 + 1^2 = 17$. The exponent goes to factors, never to terms.

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WORKED EXAMPLES

Example 1. Simplify $(r^7)^3$. Then write $(3^2)^3$ as one power of 3 and as a number.

1. $(r^7)^3$ is three copies of r^7, so there are $7 \cdot 3 = 21$ factors of r: r^{21}.
2. In the same way, $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$.

Answer: r^{21}, and $3^6 = 729$. **Check:** inside out, $3^2 = 9$ and $9^3 = 729$.

Example 2. Simplify $(-2u^4v)^3$.

1. The parentheses hold three factors: -2, u^4 and v. The cube goes to each one.
$$(-2u^4v)^3 = (-2)^3\,(u^4)^3\,v^3$$
2. $(-2)^3 = -8$: three negative factors leave one unpaired, so the result is negative.
3. $(u^4)^3$ is a power of a power, so the exponents multiply: u^{12}.

Answer: $-8u^{12}v^3$. **Check:** at $u = 2$ and $v = 1$, the left side is $(-2 \cdot 16)^3 = (-32)^3 = -32{,}768$ and the right side is $-8 \cdot 4096 = -32{,}768$.

Example 3. Simplify $\left(\dfrac{p^3}{2q^2}\right)^4$, where $q \neq 0$.

1. The fourth power goes to the top and to the bottom: $\dfrac{(p^3)^4}{(2q^2)^4}$.
2. Top: a power of a power, $(p^3)^4 = p^{12}$.
3. Bottom: a power of a product, $2^4(q^2)^4 = 16q^8$.

Answer: $\dfrac{p^{12}}{16q^8}$. **Check:** at $p = 2$ and $q = 1$, the left is $\left(\dfrac{8}{2}\right)^4 = 4^4 = 256$ and the right is $\dfrac{4096}{16} = 256$.

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