Multiplying and dividing powers

When you multiply or divide two powers of the same number, what happens to the exponents, and why?

WORDS TO KNOW

Power: a^n means n factors of a multiplied together, so $a^3 = a \cdot a \cdot a$.

Exponent: the small raised number that counts the factors, the n in a^n.

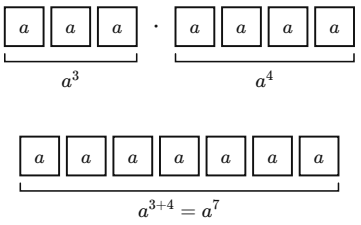
A lone factor: a base with no exponent has exponent 1, so $7 = 7^1$ and $x = x^1$.

Base: in a^n, the base is a, the number or letter being multiplied.

Same base: two powers with the same number or the same letter as base, like x^4 and x^9.

KEY IDEAS

1. Multiplying powers of the same base: add the exponents



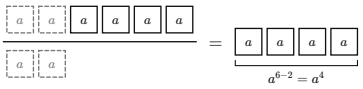
An exponent counts factors. Multiplying a^3 by a^4 puts a row of three a's next to a row of four. The rows join into one row of 3 + 4 = 7 factors, which is a^7. The base does not change.

Product of powers. For any number a and positive whole numbers m and n:

$$a^m \cdot a^n = a^{m+n}$$

It works for three or more factors too: $a^m \cdot a^n \cdot a^p = a^{m+n+p}$.

2. Dividing powers of the same base: subtract the exponents



Each factor on the bottom cancels one factor on top, because $\frac{a}{a} = 1$. In $\frac{a^6}{a^2}$, two pairs cancel and 6 − 2 = 4 factors are left: a^4.

Quotient of powers. For any nonzero number a and positive whole numbers $m > n$:

$$\frac{a^m}{a^n} = a^{m-n}$$

3. Make the bases match first

A plain number may be a power in disguise. Since $8 = 2^3$, you can write $8 \cdot 2^5$ as $2^3 \cdot 2^5 = 2^8$. If the bases are different, like x^8 and y^2, neither rule applies.

WATCH OUT

- **The base stays the same.** $9^2 \cdot 9^3 = 9^5$, not 81^5. The row is five 9's; nothing in it is an 81.
- **The bases must match.** $2^3 \cdot 5^2 = 8 \cdot 25 = 200$ cannot be written as one power of 2, 5 or 10. In the same way, x^3y^5 is already as simple as it gets.
- **Products and quotients only, never sums.** $4^3 + 4^2 = 64 + 16 = 80$, but $4^5 = 1024$. A sum is not a row of factors.
- **Dividing needs the bigger exponent on top.** In $\frac{b^2}{b^6}$ the extra four factors stay on the bottom: it is $\frac{1}{b^4}$. Do not write b^4.

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WORKED EXAMPLES

Example 1. Write $16 \cdot 2^8$ as a single power of 2, then find its value.

1. The rule needs both factors as powers of 2. Since $16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$, the product is $2^4 \cdot 2^8$.
2. Same base, so add the exponents: 4 + 8 = 12.

$$16 \cdot 2^8 = 2^4 \cdot 2^8 = 2^{12} = 4096$$

Answer: $2^{12} = 4096$. **Check:** $2^8 = 256$, and $16 \cdot 256 = 4096$.

Example 2. Simplify $\dfrac{k^6 \cdot k^8}{k^{10}}$ and $6^{11} \div 6^8$. The letter k is not zero.

1. Simplify the top first with the product rule: $k^6 \cdot k^8 = k^{14}$.
2. Now divide. Same base and $14 > 10$, so subtract: $\dfrac{k^{14}}{k^{10}} = k^{14-10} = k^4$.
3. For the second one, the base is 6 in both and $11 > 8$: $6^{11} \div 6^8 = 6^3 = 216$.

Answer: k^4 and $6^3 = 216$. **Check:** at $k = 2$, the top is $64 \cdot 256 = 16{,}384$ and $16{,}384 \div 1024 = 16 = 2^4$.

Example 3. A colony of 2^4 bacteria doubles every 30 minutes. Write the number after 5 hours as a power of 2 and as a number.

1. Five hours is 300 minutes, which is $300 \div 30 = 10$ periods of 30 minutes.
2. Each period multiplies by 2, so the ten periods multiply by 2^{10}.
3. Multiply and add the exponents: $2^4 \cdot 2^{10} = 2^{14}$.

Answer: $2^{14} = 16{,}384$ bacteria. **Check:** $2^4 = 16$ and $2^{10} = 1024$, and $16 \cdot 1024 = 16{,}384$.

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