Significant figures

A ruler, scale or stopwatch shows a number. Which of its digits did the tool really measure, and how many digits does an answer worked out from it deserve?

WORDS TO KNOW

Significant figures: the digits of a measurement that the tool actually measured.

Leading zeros: zeros before the first nonzero digit, as in 0.0480. They only place the decimal point.

Exact number: a number that was not measured: a count (6 jars), a unit definition (1 kg = 1000 g), or a number in a formula.

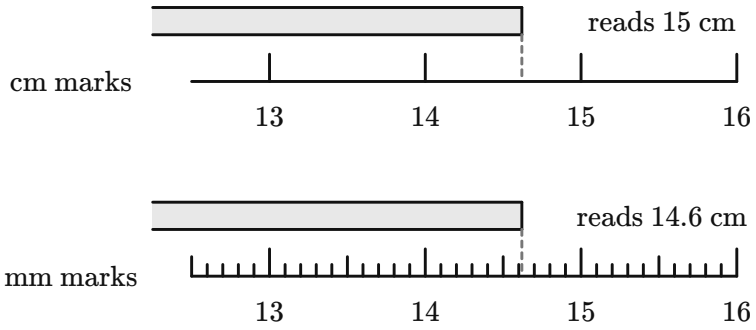
Precision of a tool: the space between its marks, or the last digit its display shows.

Trailing zeros: zeros at the end of a number, as in 1200 or 20.0.

KEY IDEAS

1. The digits show how fine the tool was

A ruler marked in whole centimeters reads an edge as 15 cm: two significant figures. A ruler with millimeter marks reads the same edge as 14.6 cm: three. The last significant digit is good to within half a unit of its place. Any digit to the right of it was never measured.



2. Which digits count

Nonzero digits always count.

Zeros between nonzero digits count: 207 g has three.

Leading zeros never count: 0.0480 kg has three (4, 8, and the last 0).

Trailing zeros after a decimal point count: 20.0°C has three.

Trailing zeros with no decimal point are unclear: 1200 cm has two, three or four.

0 . 0 0 0 4 8 0

placeholders significant

4 8 0 0 0

significant unclear

4.80×10^4

exactly three significant figures

Scientific notation settles it. Every digit written in a counts, and nothing else does. So 1.20×10^3 has exactly three significant figures.

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3. How many digits an answer deserves

These are rules of thumb, not laws. Exact numbers never limit the answer.

> **Multiply or divide:** keep as many significant figures as the measurement with the **fewest**.
>
> **Add or subtract:** keep decimal places only down to the **coarsest** last place among the measurements.
>
> **Round once, at the end.** Carry every digit through the steps.

The two rules differ because errors behave differently. In a sum, errors in the same unit add, so the coarsest place decides. In a product, errors as a fraction of the reading add, and the count of significant figures measures that fraction.

WATCH OUT

- **Exact numbers never limit.** Changing meters per second to kilometers per hour multiplies by 3.6. That 3.6 comes from unit definitions, so it does not cut the count to two.
- **Changing the unit cannot add a measured digit.** 0.0480 kg = 48.0 g: still three significant figures.
- **Do not count significant figures in a sum.** 12.4 and 3.27 both have three, but the sum is limited by the tenths place, as Example 3 shows.
- **Significant figures show precision, not accuracy.** A tape with a bent end can read 2.364 m and still be a centimeter off.

WORKED EXAMPLES

Example 1. How many significant figures? (a) 207 g (b) 0.0480 kg (c) 1200 cm

1. In (a) the 0 sits between the nonzero digits 2 and 7, so it counts: three.
2. In (b) the zeros before the 4 only place the point. The final 0 comes after the point, so it counts: three.
3. In (c) the two zeros are trailing zeros with no decimal point. The number alone cannot say.

Answer: (a) three, (b) three, (c) two, three or four. **Check:** in grams, (b) is 48.0 g, and it still has three digits.

Example 2. A phone screen measures 14.6 cm by 6.8 cm. Find its area honestly.

1. The calculator gives $14.6 \times 6.8 = 99.28$.
2. 14.6 has three significant figures and 6.8 has two. A product keeps the fewer, two.
3. Round at the ones place: 99.

Answer: about 99 cm^2. **Check:** each side could be off by 0.05 cm. The smallest area is $14.55 \times 6.75 \approx 98.2$ and the largest is $14.65 \times 6.85 \approx 100.4$, so even the ones digit is not certain.

Example 3. A hedge is measured in two parts: 12.4 m (tape marked in tenths of a meter) and 3.27 m (tape marked in centimeters). How long is it?

1. Add: $12.4 + 3.27 = 15.67$.
2. The first part reaches only tenths, so the sum keeps tenths: 15.7.

Answer: about 15.7 m. **Check:** the parts could be as short as 12.35 and 3.265, or as long as 12.45 and 3.275. So the hedge is from 15.615 to 15.725 m, and the hundredths digit could be almost anything.

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