Writing numbers in scientific notation

How do you write a very large or very small number as a number from 1 to 10 times a power of ten, and back again?

WORDS TO KNOW

Scientific notation: a number written as $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Exponent: the n on the 10. It tells how many places the decimal point moves.

Calculator display: `4.2E-6` is a calculator's shorthand for 4.2×10^{-6}. The E means "times ten to the".

Leading number: the a in front, at least 1 and less than 10. It is also called the coefficient.

Negative power of ten: $10^{-n} = \dfrac{1}{10^n}$, so $10^{-3} = 0.001$.

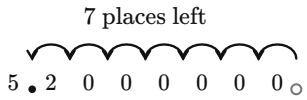
KEY IDEAS

1. Multiplying by ten moves the decimal point

Each place-value column is worth ten times the column to its right. So multiplying by 10^n moves the decimal point n places right, and dividing by 10^n moves it n places left. Every step in this lesson comes from that fact.

2. Large numbers have a positive exponent

Put the point just after the first digit to get the leading number, and drop the zeros at the end. Count how many places the point moved left. That count is the exponent.



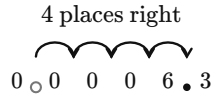
$52{,}000{,}000 = 5.2 \times 10^7$

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Quick check: a whole number with d digits has exponent $d - 1$. Here 8 digits give exponent 7.

3. Small numbers have a negative exponent

For a number between 0 and 1, put the point just after the first digit that is not zero. The point moved right, so the power of ten must be negative to undo it.



$0.00063 = 6.3 \times 10^{-4}$

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Quick check: the 6 sits in the fourth decimal place, so the exponent is −4. A number from 1 up to 10 has exponent 0, since $10^0 = 1$.

4. Why the leading number stays between 1 and 10

One number can be written many ways: 52×10^6, 5.2×10^7 and 0.52×10^8 are all equal. The rule $1 \le a < 10$ picks exactly one of them, so everyone writes the same thing, and the exponent alone tells the size.

To fix a leading number that is too big or too small, write it in scientific notation too, then combine the powers of ten. Make the leading number ten times smaller, and the exponent goes up by one.

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WATCH OUT

- **Only zeros at the end can be dropped.** In 4,080,000 the middle zero holds the 8 in place, so it is 4.08×10^6, not 4.8×10^6.
- **The exponent's sign is about size, not sign.** A negative exponent means a number between 0 and 1. A negative number keeps its minus sign in front: $-3{,}200 = -3.2 \times 10^3$.
- `4.2E-6` **is not 4.2 raised to a power.** It is $4.2 \times 10^{-6} = 0.0000042$.
- **Zero has no scientific notation.** $a \times 10^n$ is never 0, so just write 0.

WORKED EXAMPLES

Example 1. Write 150,000,000 (the distance to the Sun in km) and 4,070,000 in scientific notation.

1. For 150,000,000, the point goes just after the 1, giving 1.5. It moves 8 places left, so the exponent is 8.
2. For 4,070,000, the leading number is 4.07. The zero between 4 and 7 stays. The point moves 6 places left.

Answer: 1.5×10^8 km and 4.07×10^6. **Check:** 150,000,000 has 9 digits and 9 − 1 = 8. 4,070,000 has 7 digits and 7 − 1 = 6.

Example 2. Write 0.000306 in scientific notation, and write 8.4×10^{-3} as a decimal.

1. In 0.000306 the first digit that is not zero is the 3, in the fourth decimal place. Move the point 4 places right to get 3.06. The exponent is −4.
2. For 8.4×10^{-3}, move the point 3 places left: 8.4, then 0.84, 0.084, 0.0084. Empty places fill with zeros.

Answer: $0.000306 = 3.06 \times 10^{-4}$ and $8.4 \times 10^{-3} = 0.0084$. **Check:** 3.06 ÷ 10,000 = 0.000306.

Example 3. A student writes the population 46,300,000 as 46.3×10^6. Is it right? Write it in scientific notation.

1. The value is right: moving the point of 46.3 six places right gives 46,300,000.
2. It is not in the form, because 46.3 is not less than 10.
3. Write 46.3 = 4.63 × 10. Then regroup: $4.63 \times (10 \times 10^6) = 4.63 \times 10^7$.

Answer: 4.63×10^7. **Check:** 46,300,000 has 8 digits, and 8 − 1 = 7.

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