# Zero and negative exponents

*Dividing powers subtracts their exponents. What must $10^0$ and $10^{-3}$ mean so that this rule never breaks?*

## WORDS TO KNOW

**Zero exponent:** $a^0$, read "$a$ to the power zero".

**Negative exponent:** $a^{-n}$, read "$a$ to the power negative $n$".

**Reciprocal:** 1 divided by a number. The reciprocal of 5 is $\frac{1}{5}$, and of $\frac{2}{3}$ it is $\frac{3}{2}$. A number times its reciprocal is 1.

**Base:** the number an exponent sits on. An exponent belongs only to the base written right before it.

## KEY IDEAS

### 1. Any nonzero number to the power zero is 1

The quotient rule says $\frac{a^4}{a^4} = a^{4-4} = a^0$. But a nonzero number divided by itself is 1. For the rule to keep working, $a^0$ must be 1.

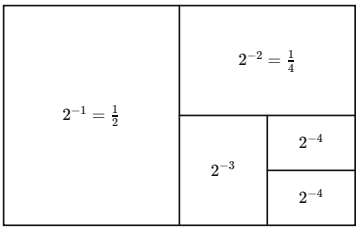
> For every nonzero $a$: $\quad a^0 = 1$
>
> $0^0$ has no value: when $a = 0$, this reasoning divides by 0.

The table agrees. Going right, each step divides by 5, and the step after $5^1$ is 5 ÷ 5 = 1.

| **Power** | $5^3$ | $5^2$ | $5^1$ | $5^0$ | $5^{-1}$ | $5^{-2}$ | $5^{-3}$ |
|---|---|---|---|---|---|---|---|
| Value | 125 | 25 | 5 | 1 | $\frac{1}{5}$ | $\frac{1}{25}$ | $\frac{1}{125}$ |

### 2. A negative exponent means "one over"

The product rule needs $a^{-n} \cdot a^n = a^0 = 1$. Two numbers whose product is 1 are reciprocals, so $a^{-n}$ is $\frac{1}{a^n}$. Halving a sheet $n$ times leaves a piece that is $2^{-n}$ of the sheet.




> For every nonzero $a$ and $b$ and whole number $n$:
>
> $$a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-n}} = a^n \qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

### 3. All the exponent rules still work

With these meanings, $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$ and $(a^m)^n = a^{mn}$ hold for every integer $m$ and $n$ (positive, negative or zero). A factor can cross the fraction bar if its exponent changes sign.

## WATCH OUT

- **A negative exponent does not make a number negative.** $6^{-2} = \frac{1}{36}$, not $-36$. A power is negative only when the base is negative and the number of factors is odd.
- **An exponent belongs to its own base.** $-4^0 = -(4^0) = -1$, but $(-4)^0 = 1$. And $4n^{-2}$ is $4/n^2$, while $(4n)^{-2}$ is $1/(16n^2)$.
- **$x^0$ is 1, not 0.** Every factor canceled, and what is left is 1.
- **Only factors cross the bar, never terms of a sum.** $\frac{1}{2^{-1} + 4^{-1}} = \frac{1}{3/4} = \frac{4}{3}$, not 2 + 4.



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## WORKED EXAMPLES

**Example 1. Find the value of each:** $(-5)^0,\ -5^0,\ (-10)^{-3},\ \left(\frac{2}{3}\right)^{-3}$.

1. The parentheses make the base −5, which is not zero, so $(-5)^0 = 1$.
2. With no parentheses, the exponent sits on the 5 only, and the minus sign comes after: $-5^0 = -(1) = -1$.
3. A negative exponent means one over the positive power. Three negative factors give a negative product.
$$(-10)^{-3} = \frac{1}{(-10)^3} = \frac{1}{-1000} = -\frac{1}{1000}$$
4. A fraction to a negative power turns over: $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{27}{8}$.

**Answer:** 1, −1, $-\frac{1}{1000}$, $\frac{27}{8}$.

**Example 2. Rewrite $\frac{4a^{-3}}{b^{-2}}$ and $\left(\frac{3}{w}\right)^{-2}$ with positive exponents. The letters are not zero.**

1. Only $a$ carries the −3, so $a^{-3} = \frac{1}{a^3}$ moves to the bottom. The 4 stays on top.
2. $\frac{1}{b^{-2}} = b^2$, so $b^2$ moves to the top.
$$\frac{4a^{-3}}{b^{-2}} = \frac{4b^2}{a^3}$$
3. Turn the fraction over and make the exponent positive: $\left(\frac{3}{w}\right)^{-2} = \left(\frac{w}{3}\right)^{2} = \frac{w^2}{9}$.

**Answer:** $\frac{4b^2}{a^3}$ and $\frac{w^2}{9}$.

**Example 3. Simplify $\frac{(g^{-3}h)^2}{g^{-5}h^4}$ with positive exponents. The letters are not zero.**

1. Top first. The square goes to each factor, and the exponents multiply: $(g^{-3}h)^2 = g^{-6}h^2$.
2. Divide base by base, subtracting exponents. Mind the signs.
$$\frac{g^{-6}h^2}{g^{-5}h^4} = g^{-6-(-5)}\,h^{2-4} = g^{-1}h^{-2}$$
3. Make the exponents positive: $g^{-1}h^{-2} = \frac{1}{gh^2}$.

**Answer:** $\frac{1}{gh^2}$. **Check:** at $g = 2$, $h = 2$: the top is $\left(\frac{2}{8}\right)^2 = \frac{1}{16}$, the bottom is $\frac{16}{32} = \frac{1}{2}$, and $\frac{1}{16} \div \frac{1}{2} = \frac{1}{8} = \frac{1}{2 \cdot 2^2}$.


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