Math 8 Unit 1 quiz 2: Monomials and the exponent rules, Writing and reading scientific notation, Comparing numbers in scientific notation (answer key)

7 questions, 22 points

ANSWERS AT A GLANCE

1. (a) $-0.3 \times 10^{10} < -2.5 \times 10^{9} < -2.05 \times 10^{9} < -7 \times 10^{8} < 1 \times 10^{6}$, that is E, A, C, B, D. (b) Glacier E.
2. (a) $14a^4b^6$ (b) $-\frac{3}{2}t^6$ (c) $-\frac{3y^4}{2x^3}$ (d) $-4x^5y^6$
3. (a) $15m^4n^3$ square meters. (b) $10m^2n^2$ meters.
4. (a) $-2m^{10}n$ (b) Yes: it is a number, -2, times powers of m and n with positive whole-number exponents, and no variable is left in a denominator.
5. Only (b) and (f) are in scientific notation. (a) 0.9 is less than 1. (c) 10 is not less than 10. (d) 2^{10} is not a power of ten. (e) the exponent 1.5 is not an integer.
6. (a) No: the country's leading part is the smaller, so the ratio is less than 10^6. (b) 5×10^5 times
7. (a) 3.004×10^{10} (b) 0.0000444 (c) 1.06×10^{-8} (d) 1,006,000,000

WORKED SOLUTIONS

1. A scientist records how the mass of ice in five glaciers changed over one year, in kilograms. A negative change is ice lost; a positive change is ice gained. (2 points)

Glacier	A	B	C	D	E
Change (kg)	-2.5×10^{9}	-7×10^{8}	-2.05×10^{9}	1×10^{6}	-0.3×10^{10}

(a) List the five changes from least to greatest.

(b) Which glacier lost the most ice?

ANSWER

(a) $-0.3 \times 10^{10} < -2.5 \times 10^{9} < -2.05 \times 10^{9} < -7 \times 10^{8} < 1 \times 10^{6}$, that is E, A, C, B, D. (b) Glacier E.

SOLUTION

(a) First rewrite E: $-0.3 \times 10^{10} = -3 \times 10^{9}$.

The only positive change, D's 1×10^{6}, is the greatest.

For the negative changes, compare their sizes and reverse the order. The sizes are 3×10^{9} (E), 2.5×10^{9} (A), 2.05×10^{9} (C) and 7×10^{8} (B). Exponent 8 is the smallest, so B's loss is the smallest; among the three with exponent 9, 3 > 2.5 > 2.05. The largest size is the least number, so

$$-0.3 \times 10^{10} < -2.5 \times 10^{9} < -2.05 \times 10^{9} < -7 \times 10^{8} < 1 \times 10^{6}$$

that is E, A, C, B, D.

(b) The most ice lost is the negative change furthest from zero, the least number: glacier E, about 3×10^{9} kg.

SCORING

1 point for rewriting E as -3×10^{9} (or otherwise placing it correctly); 1 point for the order E, A, C, B, D; 1 point for glacier E in part (b).

2. Simplify. Every variable is nonzero; write answers with positive exponents. (4 points)

(a) $(-7a^3b)(-2ab^5)$ (b) $\left(\frac{2}{3}t^5\right)\left(-\frac{9}{4}t\right)$ (c) $\frac{-24x^2y^7}{16x^5y^3}$ (d) $(-xy^2)^3(4x^2)$

ANSWER

(a) $14a^4b^6$ (b) $-\frac{3}{2}t^6$ (c) $-\frac{3y^4}{2x^3}$ (d) $-4x^5y^6$

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SOLUTION
(a) $(-7)(-2) = 14$; $a^{3+1} = a^4$; $b^{1+5} = b^6$: $14a^4b^6$.
(b) $\frac{2}{3} \cdot \left(-\frac{9}{4}\right) = -\frac{18}{12} = -\frac{3}{2}$, and $t^{5+1} = t^6$: $-\frac{3}{2}t^6$.
(c) $\frac{-24}{16} = -\frac{3}{2}$; x keeps $5 - 2 = 3$ on the bottom; y keeps $7 - 3 = 4$ on top: $-\frac{3y^4}{2x^3}$.
(d) $(-xy^2)^3 = (-1)^3x^3y^6 = -x^3y^6$. Then $(-x^3y^6)(4x^2) = -4x^5y^6$.

SCORING
1 point for each part: the coefficient, its sign and every exponent must be right.

3. A triangular garden plot has a base of $6m^3n$ meters and a height of $5mn^2$ meters, where m and n are positive. (3 points)

(a) Write the area of the plot as a monomial.

(b) A second triangular plot has the same area, and its height is $3m^2n$ meters. Find its base.

ANSWER
(a) $15m^4n^3$ square meters. (b) $10m^2n^2$ meters.

SOLUTION
(a) The area of a triangle is half the base times the height:

$$\frac{1}{2}(6m^3n)(5mn^2) = \frac{1}{2} \cdot 30m^{3+1}n^{1+2} = 15m^4n^3$$

(b) Half the base times $3m^2n$ is $15m^4n^3$, so the base times $3m^2n$ is $30m^4n^3$. Divide:

$$\text{base} = \frac{30m^4n^3}{3m^2n} = 10m^{4-2}n^{3-1} = 10m^2n^2$$

Check: $\frac{1}{2}(10m^2n^2)(3m^2n) = 15m^4n^3$.

SCORING
(a) 1 point for $15m^4n^3$. (b) 1 point for setting up the base as twice the area divided by the height (or the equivalent equation); 1 point for $10m^2n^2$.

4. Every variable is nonzero. (3 points)

(a) Simplify $\frac{(-2m^3n^{-2})^3}{4m^{-1}n^{-7}}$. Write the answer with positive exponents only.

(b) The expression is full of negative exponents. Is your simplified answer a monomial? Give the reason.

ANSWER
(a) $-2m^{10}n$ (b) Yes: it is a number, -2, times powers of m and n with positive whole-number exponents, and no variable is left in a denominator.

SOLUTION
(a) The power of the parentheses comes first. The exponent 3 reaches every factor, and $(-2)^3 = -8$ is negative because the power is odd:

$$(-2m^3n^{-2})^3 = -8m^9n^{-6}$$

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Now divide by $4m^{-1}n^{-7}$: $\frac{-8}{4} = -2$; for m, 9 − (−1) = 10; for n, −6 − (−7) = 1. So the quotient is $-2m^{10}n$.

(b) Yes. A monomial is a number times variables with positive whole-number exponents, and $-2m^{10}n$ is exactly that: both exponents came out positive, so nothing is left in a denominator.

Check (a) at $m = 1$, $n = 2$: the top is $\left(-2 \cdot \frac{1}{4}\right)^3 = -\frac{1}{8}$ and the bottom is $4 \cdot \frac{1}{128} = \frac{1}{32}$, so the left side is $-\frac{1}{8} \cdot 32 = -4$; the right side is −2 · 1 · 2 = −4.

SCORING

1 point for raising the parentheses correctly ($-8m^9n^{-6}$, or the same with positive exponents). 1 point for $-2m^{10}n$. 1 point for "yes" with a reason that names positive whole-number exponents (nothing left in a denominator).

5. Which of these are in scientific notation? For each one that is not, say why not. (3 points)

(a) 0.9×10^3 (b) 7.25×10^{-11} (c) 10×10^{-2}

(d) 4×2^{10} (e) $3.1 \times 10^{1.5}$ (f) 1×10^0

ANSWER

Only (b) and (f) are in scientific notation. (a) 0.9 is less than 1. (c) 10 is not less than 10. (d) 2^{10} is not a power of ten. (e) the exponent 1.5 is not an integer.

SOLUTION

The form is $a \times 10^n$ with $1 \leq a < 10$ and n an integer.

(a) Not: the leading number 0.9 is less than 1. (Its scientific notation is 9×10^2.)

(b) Yes: 1 ≤ 7.25 < 10, and −11 is an integer.

(c) Not: the leading number 10 is not less than 10. (It equals 1×10^{-1}.)

(d) Not: 2^{10} is a power of two, not a power of ten.

(e) Not: the exponent 1.5 is not an integer, so $10^{1.5}$ is not a place-value column.

(f) Yes: 1 is allowed as a leading number and 0 is an integer. It is the number 1.

SCORING

$\frac{1}{2}$ point for each of (a) to (f): the right decision, with the reason for each one that is not in the form.

6. A beekeeper's hive holds about 4×10^4 honeybees. A survey estimates that a whole country has about 2×10^{10} honeybees. Omar says that since the exponents differ by 6, the country must have at least 10^6 times as many honeybees as one hive. (3 points)

(a) Without dividing, explain whether Omar is right.

(b) How many times as many honeybees as one hive does the country have? Give your answer in scientific notation.

ANSWER

(a) No: the country's leading part is the smaller, so the ratio is less than 10^6. (b) 5×10^5 times

SOLUTION

(a) Omar is wrong. The ratio is the quotient of the leading parts times 10^6. The country's leading part, 2, is smaller than the hive's, 4, so the quotient of the leading parts is less than 1 and the ratio is less

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than 10^6.

(b) Divide the leading parts and subtract the exponents:

$$\frac{2 \times 10^{10}}{4 \times 10^4} = \frac{2}{4} \times 10^6$$

The leading part $\frac{2}{4}$ is less than 1, so multiply it by 10 and divide the power of ten by 10:

$$\frac{2}{4} \times 10^6 = 5 \times 10^5$$

The country has 5×10^5 times as many honeybees as one hive. Check: the exponents differ by 6 and the country's leading part is the smaller, so the ratio had to be below 10^6; it is.

SCORING

1 point for (a): Omar is wrong, because the country's leading part is the smaller; 1 point for dividing the leading parts and subtracting the exponents; 1 point for rewriting the result as 5×10^5.

7. Each row of the table names one number in two ways. Fill in every empty cell.

Row	Ordinary number	Scientific notation
(a)	30,040,000,000	_____
(b)	_____	4.44×10^{-5}
(c)	0.0000000106	_____
(d)	_____	1.006×10^9

(4 points)

ANSWER

(a) 3.004×10^{10} (b) 0.0000444 (c) 1.06×10^{-8} (d) 1,006,000,000

SOLUTION

(a) The leading number is 3.004; the two zeros between 3 and 4 stay, and only the trailing zeros go. The number has 11 digits, so the exponent is 10: 3.004×10^{10}.

(b) The point moves five places left, so the first 4 lands in the fifth decimal place, after four zeros: 0.0000444.

(c) The first digit that is not zero, the 1, is in the eighth decimal place, so the exponent is −8. The zero between 1 and 6 stays: 1.06×10^{-8}.

(d) The point moves nine places right. The 0, 0 and 6 use up three moves, and six zeros fill the rest: 1,006,000,000. Check: 10 digits, one more than the exponent.

SCORING

1 point for each row (a) to (d). No point for a value in the form $a \times 10^n$ whose leading number is not from 1 up to but not including 10.

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