Math 8 Unit 1 quiz 3: Calculating in scientific notation, Significant figures (answer key)

8 questions, 22 points

ANSWERS AT A GLANCE

1. 3.72×10^{7} rides
2. (a) 6×10^{8} (b) 1.25×10^{-10}
3. (a) About 1.04×10^{-2} meters. (b) About 2.5×10^{7} seconds.
4. (a) Every operation is a multiplication, which may be reordered and regrouped. (b) 1.2×10^{-11}, shown as 1.2E-11.
5. (a) 60.00 s (b) four (c) 1.000 min
6. (a) About 7 cm, one significant figure. (b) From 6.15 cm to 7.25 cm.
7. 12.0 km per liter
8. (a) She used the rule for products (fewest significant figures); a sum follows the coarsest last place. (b) 126.1 g (c) The total could be off by up to 0.055 g, so tenths are supported and tens are far too coarse.

WORKED SOLUTIONS

1. In one year a city counted 3.2×10^{7} bus rides and 5.2×10^{6} tram rides. How many rides were there in all? Give the answer in scientific notation. (2 points)

ANSWER

3.72×10^{7} rides

SOLUTION

The powers differ, so rewrite the tram rides with the larger power. The exponent goes up by 1, so the leading part is divided by 10: $5.2 \times 10^{6} = 0.52 \times 10^{7}$. Then

$$(3.2 + 0.52) \times 10^{7} = 3.72 \times 10^{7}$$

rides in all.

SCORING

1 point for writing both numbers with the same power of ten. 1 point for the correct sum in scientific notation.

2. Calculate. Give each answer in scientific notation.

(a) $(2 \times 10^{-3})(7.5 \times 10^{8})(4 \times 10^{2})$ (b) $(5 \times 10^{-4})^{3}$ (3 points)

ANSWER

(a) 6×10^{8} (b) 1.25×10^{-10}

SOLUTION

(a) The leading parts give 2 × 7.5 × 4 = 60 and the exponents give −3 + 8 + 2 = 7, so the product is $60 \times 10^{7} = 6 \times 10^{8}$.

(b) The cube is three equal factors: 5 × 5 × 5 = 125 and 3 × (−4) = −12, so it is 125×10^{-12}. Write $125 = 1.25 \times 10^{2}$: the answer is 1.25×10^{-10}.

SCORING

1 point for (a), with the leading part 60 renormalized. 1 point for multiplying the leading parts in (b) to 125 and the exponents to −12. 1 point for renormalizing (b) to 1.25×10^{-10}.

3. A hair on a person's head grows about 4×10^{-9} meters each second. (3 points)

(a) A 30-day month is about 2.6×10^{6} seconds. About how much does the hair grow in a month? Give the answer in meters, in scientific notation.

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(b) About how many seconds does the hair take to grow 1×10^{-1} meters? Give the answer in scientific notation.

ANSWER

(a) About 1.04×10^{-2} meters. (b) About 2.5×10^{7} seconds.

SOLUTION

(a) Growth is the rate times the time. The leading parts give $4 \times 2.6 = 10.4$ and the exponents give $-9 + 6 = -3$:

$$\begin{aligned}(4 \times 10^{-9})(2.6 \times 10^{6}) &= 10.4 \times 10^{-3} \\ &= 1.04 \times 10^{-2}\ \text{m}\end{aligned}$$

That is about a centimeter a month.

(b) The time is the length divided by the growth each second:

$$\begin{aligned}\frac{1 \times 10^{-1}}{4 \times 10^{-9}} &= 0.25 \times 10^{-1-(-9)} \\ &= 0.25 \times 10^{8} \\ &= 2.5 \times 10^{7}\ \text{s}\end{aligned}$$

That is a little under ten months.

SCORING

1 point for (a), 10.4×10^{-3} renormalized to 1.04×10^{-2}. 1 point for setting up (b) as the length divided by the growth each second, with the exponent $-1 - (-9) = 8$. 1 point for renormalizing (b) to 2.5×10^{7}.

4. Mateo needs the product $(6 \times 10^{-4})(2 \times 10^{-8})$. He types it into a calculator as $6 \times 10^{-4} \times 2 \times 10^{-8}$, with no parentheses. (3 points)

(a) Using the order of operations, explain why the missing parentheses do not change his answer here.

(b) Find the product in scientific notation, and write how the calculator shows it with the letter E.

ANSWER

(a) Every operation is a multiplication, which may be reordered and regrouped. (b) 1.2×10^{-11}, shown as 1.2E-11.

SOLUTION

(a) Typed this way there are four numbers joined only by $\times$. The calculator multiplies from left to right, and multiplication may be reordered and regrouped, so any grouping gives the same product as $(6 \times 10^{-4})(2 \times 10^{-8})$. Parentheses matter for a division, where the order changes which numbers are in the divisor.

(b) The leading parts give $6 \times 2 = 12$ and the exponents give $-4 + (-8) = -12$:

$$12 \times 10^{-12} = 1.2 \times 10^{-11}$$

The calculator shows 1.2E-11.

SCORING

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1 point for (a): the only operation is multiplication, so order and grouping do not matter. 1 point for 1.2×10^{-11}, renormalized. 1 point for the display 1.2E-11.

5. A timer shows only seconds, to the hundredth of a second, and never switches to minutes. It stops at exactly one minute. (3 points)

(a) Write the reading as the timer shows it, in seconds.

(b) How many significant figures does the reading have?

(c) Write the same reading in minutes, keeping its significant figures.

ANSWER

(a) 60.00 s (b) four (c) 1.000 min

SOLUTION

(a) The timer shows hundredths, so it shows two digits after the point even when they are zeros: 60.00 s.

(b) Four. The 6 counts, and the three zeros that follow are a trailing zero in the ones place and two after the decimal point, all shown by the timer.

(c) 1 min = 60 s is a unit definition, so it is exact. Dividing by 60 moves the reading into minutes without changing its significant figures: 60.00 ÷ 60 = 1.000 min, four figures.

SCORING

1 point for 60.00 s with both decimal zeros. 1 point for four. 1 point for 1.000 min (an answer of 1 min or 1.0 min earns 0).

6. On Monday a bean plant is 14.3 cm tall, measured with a ruler marked in millimeters. By Friday it is too tall for the ruler, and a tape marked only in whole centimeters reads 21 cm. (3 points)

(a) How much did the plant grow, reported honestly? How many significant figures does your answer have?

(b) Using the half-unit error of each reading, find the smallest and largest growth the readings allow.

ANSWER

(a) About 7 cm, one significant figure. (b) From 6.15 cm to 7.25 cm.

SOLUTION

(a) 21 − 14.3 = 6.7. In a difference the coarsest last place decides: the tape reading reaches only the ones place, so the growth is reported to the ones: about 7 cm, which has one significant figure.

(b) The Monday height is between 14.25 and 14.35 cm; the Friday height is between 20.5 and 21.5 cm. The smallest growth takes the smallest Friday height and the largest Monday height: 20.5 − 14.35 = 6.15 cm. The largest takes the opposite: 21.5 − 14.25 = 7.25 cm. A range more than a centimeter wide is why only the ones digit is reported.

SCORING

(a) 1 point for 7 cm (or about 7 cm) with one significant figure; 6.7 cm earns 0. (b) 1 point for pairing the extremes correctly (smallest Friday with largest Monday, and the reverse), 1 point for 6.15 and 7.25 cm.

7. Hannah's family fills the car's tank, drives until the trip meter shows 518 km (it shows whole kilometers), and then refills it with 43.06 L of fuel. Find how many kilometers the car went on one liter of fuel, to an honest number of significant figures. (2 points)

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ANSWER
12.0 km per liter

SOLUTION
Divide the distance by the fuel: the calculator shows

$$518 \div 43.06 = 12.0297$$

The distance has three significant figures and the fuel four. A quotient keeps as many as the fewest, three: 12.0 km per liter. A final zero is written when it is the third figure.

SCORING
1 point for dividing to get the calculator value. 1 point for rounding to three significant figures, with any final zero written.

8. Hannah weighs a beaker of water at 125.6 g on a balance that reads tenths of a gram. She adds 0.48 g of salt, weighed on a balance that reads hundredths. She reports the total as 1.3×10^2 g, because "0.48 has only two significant figures." (3 points)

(a) What rule did Hannah use, and which rule applies instead?

(b) Report the total honestly.

(c) Use the greatest possible error of each reading to explain why your answer is the right size.

ANSWER
(a) She used the rule for products (fewest significant figures); a sum follows the coarsest last place. (b) 126.1 g (c) The total could be off by up to 0.055 g, so tenths are supported and tens are far too coarse.

SOLUTION
(a) Counting significant figures is the rule for multiplying and dividing. For adding, the coarsest last place among the readings decides, here the tenths place of 125.6.

(b) 125.6 + 0.48 = 126.08, kept to tenths: 126.1 g.

(c) The readings can be off by up to 0.05 g and 0.005 g. In a sum these add, so the total is off by at most 0.055 g: the tenths digit carries real information but the hundredths does not. Hannah's 1.3×10^2 rounds to the nearest 10 g, throwing away digits the balance measured. The salt is small, so its relative error is large, but in a sum errors are compared in grams, not as fractions.

SCORING
1 point for naming her rule as the product rule and the place rule for sums. 1 point for 126.1 g. 1 point for the error argument: errors of 0.05 and 0.005 g add to 0.055 g, so tenths are supported.

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