Math 8 Unit 1 worksheet: Comparing numbers in scientific notation (answer key)

ANSWERS AT A GLANCE

1. (a) $<$ (b) $>$ (c) $>$ (d) $=$ (e) $>$ (f) $<$ (g) $<$ (h) $>$
2. (a) Hummingbird 2×10^{-2}, housefly 5×10^{-3}, shutter 8×10^{-3}, blink 3×10^{-1}, light 1.5×10^{-8}. Shortest to longest: light, housefly, shutter, hummingbird, blink. (b) About 60 times.
3. 2×10^5 times
4. (a) Between 10^2 and 10^3. (b) $\dfrac{3 \times 10^{12}}{8 \times 10^9} = 375$, so roughly 400 trees per person.
5. (a) $n = -2, -1, 0, 1, 2, 3, 4$ (b) $d = 1, 2, 3$
6. (a) For example 1.5×10^6 and 9×10^5: the first is less than twice the second. (b) 99 times: yes, for example 9.9×10^6 and 1×10^5. 100 times: no, because the first is less than 10^7 and the second is at least 10^5, so the ratio is less than $\dfrac{10^7}{10^5} = 100$.

WORKED SOLUTIONS

1. Write $<$, $>$ or $=$ in each blank. Rewrite any number that is not in scientific notation first.

(a) 7.1×10^5 ☐ 2.3×10^6

(b) 4.6×10^{-7} ☐ 4.06×10^{-7}

(c) 1.5×10^{-2} ☐ 9.5×10^{-3}

(d) 3.02×10^4 ☐ $30{,}200$

(e) 0.0061 ☐ 6.1×10^{-4}

(f) 52×10^3 ☐ 5.3×10^4

(g) -3.5×10^2 ☐ -8×10^1

(h) 9.1×10^{-1} ☐ 0.09×10^1

ANSWER

(a) $<$ (b) $>$ (c) $>$ (d) $=$ (e) $>$ (f) $<$ (g) $<$ (h) $>$

SOLUTION

(a) The exponents are 5 and 6, and 5 < 6, so $7.1 \times 10^5 < 2.3 \times 10^6$.

(b) Both exponents are −7, so the leading parts decide: 4.60 > 4.06, so $4.6 \times 10^{-7} > 4.06 \times 10^{-7}$.

(c) The exponents are −2 and −3, and −2 > −3, so $1.5 \times 10^{-2} > 9.5 \times 10^{-3}$, even though 9.5 > 1.5.

(d) $30{,}200 = 3.02 \times 10^4$, so the two are equal.

(e) $0.0061 = 6.1 \times 10^{-3}$, and −3 > −4, so $0.0061 > 6.1 \times 10^{-4}$.

(f) $52 \times 10^3 = 5.2 \times 10^4$. The exponents match and 5.2 < 5.3, so $52 \times 10^3 < 5.3 \times 10^4$.

(g) These are negative: $-3.5 \times 10^2 = -350$ and $-8 \times 10^1 = -80$. The one further from zero is smaller, so $-3.5 \times 10^2 < -8 \times 10^1$.

(h) $0.09 \times 10^1 = 9 \times 10^{-1}$. The exponents match and 9.1 > 9.0, so $9.1 \times 10^{-1} > 0.09 \times 10^1$.

2. The table gives some very short times.

Event	Time (seconds)
one beat of a hummingbird's wings	0.02
one beat of a housefly's wings	0.5×10^{-2}
a camera shutter opening and closing	8×10^{-3}
a blink of an eye	3×10^{-1}
light crossing a classroom	15×10^{-9}

(a) Write each time in scientific notation. Then list the events from shortest to longest.

(b) About how many times as long as one beat of a housefly's wings is a blink of an eye?

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ANSWER

(a) Hummingbird 2×10^{-2}, housefly 5×10^{-3}, shutter 8×10^{-3}, blink 3×10^{-1}, light 1.5×10^{-8}. Shortest to longest: light, housefly, shutter, hummingbird, blink. (b) About 60 times.

SOLUTION

(a) Rewrite the three that are not in scientific notation: $0.02 = 2 \times 10^{-2}$; $0.5 \times 10^{-2} = 5 \times 10^{-3}$ (leading part times 10, power of ten divided by 10); $15 \times 10^{-9} = 1.5 \times 10^{-8}$.

Sort by exponent: -8 (light), -3 (housefly and shutter), -2 (hummingbird), -1 (blink). In the group with exponent -3, $5 < 8$, so the housefly's wingbeat is shorter than the shutter.

Shortest to longest: light, housefly, shutter, hummingbird, blink. As first printed, the housefly's exponent -2 would have put it after the shutter.

(b) Divide the leading parts and subtract the exponents:

$$\frac{3 \times 10^{-1}}{5 \times 10^{-3}} = \frac{3}{5} \times 10^{2} = 0.6 \times 100 = 60$$

A blink lasts about 60 times as long as one beat of a housefly's wings.

3. A city reservoir holds about 4×10^{10} liters of water. The town's outdoor swimming pool holds about 2×10^{5} liters. How many times as much water as the pool does the reservoir hold? Give your answer in scientific notation.

ANSWER

2×10^{5} times

SOLUTION

Divide the leading parts and subtract the exponents:

$$\frac{4 \times 10^{10}}{2 \times 10^{5}} = \frac{4}{2} \times 10^{10-5} = 2 \times 10^{5}$$

The leading part 2 is from 1 up to 10, so this is already in scientific notation. The reservoir holds 2×10^{5} times as much water as the pool.

Check with the exponents: they differ by 5 and the reservoir's leading part is the larger, so the ratio had to be at least 10^{5} and less than 10^{5+1}. It is.

4. There are about 3.04×10^{12} trees on Earth and about 8.1×10^{9} people.

(a) Without dividing, between which two powers of ten must the number of trees per person lie? Explain.

(b) Round each leading part to one digit and estimate the number of trees per person.

ANSWER

(a) Between 10^{2} and 10^{3}. (b) $\frac{3 \times 10^{12}}{8 \times 10^{9}} = 375$, so roughly 400 trees per person.

SOLUTION

(a) The exponents are 12 and 9, which differ by 3, so the ratio is more than 10^{2} and less than 10^{4}. The larger number, the trees, has the smaller leading part ($3.04 < 8.1$), so the ratio is below 10^{3}. It lies between 10^{2} and 10^{3}: more than 100 and fewer than 1000 trees per person.

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(b) To one digit, 3.04 rounds to 3 and 8.1 rounds to 8:

$$\frac{3 \times 10^{12}}{8 \times 10^{9}} = \frac{3}{8} \times 10^{3} = 0.375 \times 1000 = 375$$

That is roughly 400 trees per person, and it lies between 10^2 and 10^3 as part (a) promised.

5. (a) Find every integer n for which 3.6×10^{n} is greater than 0.0052 and less than 81,000.

(b) Find every digit d from 1 to 9 for which $d.5 \times 10^{-3}$ is less than 0.00407.

ANSWER

(a) $n = -2, -1, 0, 1, 2, 3, 4$ (b) $d = 1, 2, 3$

SOLUTION

(a) Write the two limits in scientific notation: $0.0052 = 5.2 \times 10^{-3}$ and $81{,}000 = 8.1 \times 10^{4}$.

Lower limit: with $n = -3$ the exponents match and 3.6 < 5.2, so 3.6×10^{-3} is too small. Any $n \geq -2$ has the larger exponent, so it is greater than 5.2×10^{-3}.

Upper limit: with $n = 4$ the exponents match and 3.6 < 8.1, so $3.6 \times 10^{4} = 36{,}000$ is less than 81,000. With $n = 5$ the exponent is larger, so it is too big.

So $n = -2, -1, 0, 1, 2, 3, 4$.

(b) $0.00407 = 4.07 \times 10^{-3}$. The exponents match, so the leading parts decide: $d.5 < 4.07$. That holds for 1.5, 2.5 and 3.5, but not for 4.5 or more. So $d = 1, 2, 3$.

6. Maya says: "If one number in scientific notation has exponent 6 and another has exponent 5, the first is always at least 10 times the second."

(a) Give two such numbers that show Maya is wrong.

(b) Can the first number be 99 times the second? Can it be 100 times the second? For each, give an example or explain why not.

ANSWER

(a) For example 1.5×10^{6} and 9×10^{5}: the first is less than twice the second. (b) 99 times: yes, for example 9.9×10^{6} and 1×10^{5}. 100 times: no, because the first is less than 10^{7} and the second is at least 10^{5}, so the ratio is less than $\frac{10^{7}}{10^{5}} = 100$.

SOLUTION

(a) Take $1.5 \times 10^{6} = 1{,}500{,}000$ and $9 \times 10^{5} = 900{,}000$. Then

$$\frac{1.5 \times 10^{6}}{9 \times 10^{5}} = \frac{1.5}{9} \times 10 \approx 1.7$$

so the first is less than twice the second, far from 10 times. Any pair where the second leading part is larger than the first works the same way, since then the ratio is below 10.

(b) To make the ratio as large as possible, give the first number a large leading part and the second a small one. With 9.9×10^{6} and 1×10^{5}:

$$\frac{9.9 \times 10^{6}}{1 \times 10^{5}} = 9.9 \times 10^{1} = 99$$

So 99 times is possible.

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100 times is not. A leading part a has $1 \leq a < 10$, so a number with exponent 6 is less than 10^7, and a number with exponent 5 is at least 10^5. A smaller top and a larger bottom both shrink a fraction, so the ratio is always less than $\frac{10^7}{10^5} = 10^2 = 100$. It can come as close to 100 as we like, but never reach it.

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