Math 8 Unit 1 worksheet: Monomials and the exponent rules (answer key)

ANSWERS AT A GLANCE

1. (a) $-30p^9$ (b) $28w^6$ (c) $-6r^3s^5$ (d) $4k^7$ (e) $-4a^4b^2c$ (f) $-10g^4h^2$ (g) $\frac{2}{3}m^6n$ (h) $-6x^3y^6$

2. (a) $-7k^8$ (b) $\frac{2s^4}{3}$ (c) $\frac{5b^4}{2a^4}$ (d) $\frac{2m^3}{5n^2p^2}$

3. (a) $-24a^7b^7$ (b) $5m^7n^2$ (c) $\frac{4x^2}{y^3}$ (d) $-3p^4$

4. (a) $\frac{15x^4}{y^3}$ (b) $-\frac{3vw^2}{4u^3}$ (c) $\frac{m}{9}$ (d) $\frac{x^2}{2}$

5. (a) $18x^2$ square feet (b) 162 square feet

6. (a) $PQ = 48x^7y^3$, $\frac{P}{Q} = 3x^3y$; both are monomials. (b) $PQ = 54a^6b$ is a monomial; $\frac{P}{Q} = \frac{2}{3b}$ is not. (c) $PQ = 50m^4n^4$, $\frac{P}{Q} = 2n^4$; both are monomials.

7. (a) $12x^3y$ square centimeters. (b) Length $36x^2$, width $\frac{9}{2}xy$, area $162x^3y$ square centimeters. (c) $\frac{27}{2} = 13.5$ times, for every positive x and y.

WORKED SOLUTIONS

1. Multiply. Write each answer as a single monomial.

(a) $(6p^2)(-5p^7)$ (b) $(-4w)(-7w^5)$ (c) $(-3r^2s)(2rs^4)$ (d) $(0.5k^3)(8k^4)$

(e) $(4ab^2)(-a^3c)$ (f) $(-12gh^2)\left(\frac{5}{6}g^3\right)$ (g) $\left(\frac{3}{5}m^4\right)\left(\frac{10}{9}m^2n\right)$ (h) $(-x^2)(-3xy)(-2y^5)$

ANSWER

(a) $-30p^9$ (b) $28w^6$ (c) $-6r^3s^5$ (d) $4k^7$ (e) $-4a^4b^2c$ (f) $-10g^4h^2$ (g) $\frac{2}{3}m^6n$ (h) $-6x^3y^6$

SOLUTION

Multiply the coefficients with their signs; for each variable, add its exponents.

(a) $6 \cdot (-5) = -30$ and $2 + 7 = 9$: $-30p^9$.

(b) $(-4)(-7) = 28$, and w is w^1, so $1 + 5 = 6$: $28w^6$.

(c) $(-3)(2) = -6$; for r, $2 + 1 = 3$; for s, $1 + 4 = 5$: $-6r^3s^5$.

(d) $0.5 \cdot 8 = 4$ and $3 + 4 = 7$: $4k^7$.

(e) The second coefficient is -1, so $4 \cdot (-1) = -4$. For a, $1 + 3 = 4$; b^2 is only in the first factor and c only in the second, so both pass through: $-4a^4b^2c$.

(f) $(-12) \cdot \frac{5}{6} = -10$; for g, $1 + 3 = 4$; h^2 is only in the first factor and passes through: $-10g^4h^2$.

(g) $\frac{3}{5} \cdot \frac{10}{9} = \frac{30}{45} = \frac{2}{3}$; for m, $4 + 2 = 6$; n passes through: $\frac{2}{3}m^6n$.

(h) Three negative coefficients: $(-1)(-3)(-2) = -6$. For x, $2 + 1 = 3$; for y, $1 + 5 = 6$: $-6x^3y^6$.

2. Divide. Every variable is nonzero. Write each answer with positive exponents and the coefficient in lowest terms.

(a) $\frac{56k^{10}}{-8k^2}$ (b) $\frac{14r^3s^6}{21r^3s^2}$ (c) $\frac{-30a^2b^5}{-12a^6b}$ (d) $\frac{20m^4n}{50mn^3p^2}$

ANSWER

(a) $-7k^8$ (b) $\frac{2s^4}{3}$ (c) $\frac{5b^4}{2a^4}$ (d) $\frac{2m^3}{5n^2p^2}$

SOLUTION

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Divide the coefficients; for each variable, subtract the smaller exponent from the larger and keep what is left on the side that had more.

(a) $\frac{56}{-8} = -7$ and 10 − 2 = 8 on top: $-7k^8$.

(b) $\frac{14}{21} = \frac{2}{3}$. The powers of r are equal, so they cancel to 1. For s, 6 − 2 = 4 on top: $\frac{2s^4}{3}$.

(c) $\frac{-30}{-12} = \frac{5}{2}$. For a the bottom has more, 6 − 2 = 4, so a^4 stays on the bottom. For b the top has more, 5 − 1 = 4: $\frac{5b^4}{2a^4}$.

(d) $\frac{20}{50} = \frac{2}{5}$. For m, 4 − 1 = 3 on top. For n, 3 − 1 = 2 on the bottom. The p^2 is only on the bottom and stays there: $\frac{2m^3}{5n^2p^2}$.

3. Simplify. Every variable is nonzero. Work out each power of a product in parentheses first.

(a) $(-2a^2b)^3(3ab^4)$ (b) $(5m^3)(-m^2n)^2$ (c) $\frac{(6x^3y)^2}{9x^4y^5}$ (d) $\frac{(-3p^2)^3}{(3p)^2}$

ANSWER

(a) $-24a^7b^7$ (b) $5m^7n^2$ (c) $\frac{4x^2}{y^3}$ (d) $-3p^4$

SOLUTION

(a) $(-2a^2b)^3 = (-2)^3a^6b^3 = -8a^6b^3$. Then $(-8a^6b^3)(3ab^4) = -24a^7b^7$.

(b) $(-m^2n)^2 = (-1)^2m^4n^2 = m^4n^2$, positive because the power is even. Then $(5m^3)(m^4n^2) = 5m^7n^2$.

(c) $(6x^3y)^2 = 36x^6y^2$. Then $\frac{36x^6y^2}{9x^4y^5}$: $\frac{36}{9} = 4$, x keeps 6 − 4 = 2 on top, y keeps 5 − 2 = 3 on the bottom: $\frac{4x^2}{y^3}$.

(d) $(-3p^2)^3 = -27p^6$, negative because the power is odd, and $(3p)^2 = 9p^2$. Then $\frac{-27p^6}{9p^2} = -3p^4$.

4. Simplify. Every variable is nonzero. Write each answer with positive exponents only.

(a) $(5x^{-2}y)(3x^6y^{-4})$ (b) $\frac{-9u^{-4}v}{12u^{-1}w^{-2}}$ (c) $(3m^{-2})^{-2} \cdot m^{-3}$ (d) $\frac{(2x^3)^{-1}}{x^{-5}y^0}$

ANSWER

(a) $\frac{15x^4}{y^3}$ (b) $-\frac{3vw^2}{4u^3}$ (c) $\frac{m}{9}$ (d) $\frac{x^2}{2}$

SOLUTION

(a) 5 · 3 = 15; for x, −2 + 6 = 4; for y, 1 + (−4) = −3. So $15x^4y^{-3} = \frac{15x^4}{y^3}$.

(b) $\frac{-9}{12} = -\frac{3}{4}$; for u, −4 − (−1) = −3; v is only on top and stays; w^{-2} is only on the bottom and crosses up as w^2. So $-\frac{3}{4}u^{-3}vw^2 = -\frac{3vw^2}{4u^3}$.

(c) $(3m^{-2})^{-2} = 3^{-2}m^{(-2)(-2)} = \frac{1}{9}m^4$. Then $\frac{1}{9}m^4 \cdot m^{-3} = \frac{1}{9}m^1 = \frac{m}{9}$.

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(d) $y^0 = 1$, and $(2x^3)^{-1} = \frac{1}{2}x^{-3}$. Dividing by x^{-5} subtracts exponents: $-3 - (-5) = 2$. So the result is $\frac{1}{2}x^2 = \frac{x^2}{2}$.

5. Hannah's bedroom floor is a rectangle $4x$ feet by $5x$ feet. A closet in one corner, $2x$ feet by x feet, is part of that rectangle and does not get new flooring.

(a) Write the area that gets new flooring as a monomial.

(b) Find that area when $x = 3$.

ANSWER

(a) $18x^2$ square feet (b) 162 square feet

SOLUTION

(a) The whole rectangle has area $(4x)(5x) = 20x^2$, since the coefficients multiply and $x \cdot x = x^2$. The closet has area $(2x)(x) = 2x^2$. The two areas are like terms, so subtract the coefficients:

$$20x^2 - 2x^2 = 18x^2$$

The new flooring covers $18x^2$ square feet.

(b) At $x = 3$: $18 \cdot 3^2 = 18 \cdot 9 = 162$ square feet.

Check with the lengths: the room is 12 ft by 15 ft, or 180 square feet, and the closet is 6 ft by 3 ft, or 18 square feet; 180 − 18 = 162.

6. For each pair of monomials P and Q, find the product PQ and the quotient $\frac{P}{Q}$, simplified. Then say whether each result is a monomial. Every variable is nonzero.

(a) $P = 12x^5y^2,\ Q = 4x^2y$

(b) $P = 6a^3,\ Q = 9a^3b$

(c) $P = -10m^2n^4,\ Q = -5m^2$

ANSWER

(a) $PQ = 48x^7y^3$, $\frac{P}{Q} = 3x^3y$; both are monomials. (b) $PQ = 54a^6b$ is a monomial; $\frac{P}{Q} = \frac{2}{3b}$ is not. (c) $PQ = 50m^4n^4$, $\frac{P}{Q} = 2n^4$; both are monomials.

SOLUTION

(a) $PQ = (12 \cdot 4)x^{5+2}y^{2+1} = 48x^7y^3$. $\frac{P}{Q} = \frac{12}{4}x^{5-2}y^{2-1} = 3x^3y$. Both are a number times powers of variables with positive whole exponents, so both are monomials.

(b) $PQ = (6 \cdot 9)a^{3+3}b = 54a^6b$, a monomial. For the quotient, $\frac{6}{9} = \frac{2}{3}$, the powers of a are equal and cancel, and b is only in the divisor, so it stays on the bottom: $\frac{P}{Q} = \frac{2}{3b}$. A variable is left in the denominator, so this is not a monomial.

(c) $PQ = (-10)(-5)m^{2+2}n^4 = 50m^4n^4$. $\frac{P}{Q} = \frac{-10}{-5}m^{2-2}n^4 = 2n^4$, since the powers of m cancel. Both are monomials: every variable of Q appears in P with at least as large an exponent.

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7. A rectangular banner is $4x^2$ centimeters long and $3xy$ centimeters wide, where x and y are positive.

(a) Write the area of the banner as a monomial.

(b) A second banner is designed by putting $3x$ in place of x and $\dfrac{y}{2}$ in place of y in both lengths. Write its length, its width and its area as monomials.

(c) How many times the area of the first banner is the area of the second? Does the answer depend on x and y?

ANSWER

(a) $12x^3y$ square centimeters. (b) Length $36x^2$, width $\dfrac{9}{2}xy$, area $162x^3y$ square centimeters. (c) $\dfrac{27}{2} = 13.5$ times, for every positive x and y.

SOLUTION

(a) $(4x^2)(3xy) = 12x^{2+1}y = 12x^3y$ square centimeters.

(b) Length: $4(3x)^2 = 4 \cdot 9x^2 = 36x^2$. Width: $3(3x)\left(\dfrac{y}{2}\right) = \dfrac{9}{2}xy$. Area: $(36x^2)\left(\dfrac{9}{2}xy\right) = 162x^3y$ square centimeters.

(c) $\dfrac{162x^3y}{12x^3y} = \dfrac{162}{12} = \dfrac{27}{2}$, since the powers of x and of y cancel. The second banner has 13.5 times the area, whatever x and y are. It is $3^2 \cdot 3 \cdot \dfrac{1}{2}$: x appears three times in the area, and y once.

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